

I. 填充題：請將答案按字母順序寫在答案紙前八行。不要寫計算過程

(每格 8 分)

- The function  $f(x) = -35x - x^5$  is one-to-one. Calculate  $(f^{-1})'(-36) = \underline{A}$ .
- $\int (\ell_n x^{20})^2 dx = \underline{B}$ .
- $\lim_{x \rightarrow 0^+} (\cosh x + 5x)^{\frac{1}{2x}} = \underline{C}$ .
- Let  $f$  be a differentiable function of  $x$  and  $y$  and  $z$ , and  $g$  is a differentiable function of a single variable. Let  $h(x, y, z) = g(f(x, y, z))$ . Compute  $h_z(x, y, z) = \underline{D}$ .
- Find the radius of convergence of  $\sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^2} (x+1)^k = \underline{E}$ .
- $\int \frac{dx}{x^2 \sqrt{x^2 - 4}} = \underline{F}$ .
- The maximum of the function  $f(x, y) = 5xy$  on the unit circle  $x^2 + y^2 = 1$  is  $\underline{G}$ .
- $H(x) = \int_{\sqrt{x}}^{x^2} \frac{1}{2+t^2} dt$ . Calculate  $H'(2) = \underline{H}$ .

II. 計算與證明：請詳細寫出每一推導步驟

$$1. \text{ Let } f(x, y) = \begin{cases} \frac{xy + y^3}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

(i) Does  $f_x(0, 0), f_y(0, 0)$  exist? (8 分)

(ii) Is  $f$  continuous at  $(0, 0)$ ? (8 分)

2. Let  $a_k = \left(1 + \frac{1}{k^2}\right)^k, \forall k \in \mathbb{N}$ . Determine the convergence or divergence of the

series  $\sum_{k=1}^{\infty} (a_{k+1} - a_k)$ . (10 分)

3. Find the tangent line of the curve  $(x^2 + y^2)^2 = (x - y)^2$  at  $(1, -1)$ . (10 分)