國　立　清　華　大　學　命　題　紙

九十二學年度 科技管理 系（所） 丙 組碩士班研究生招生考試

科目 微積分 科號 5903 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. **填充題：請將答案按字母順序寫在答案紙前八行。不要寫計算過程。**
   （每格 8 分）

   (a) $\lim_{x\to\infty}(e^x+x)^{1/x} = \underline{(A)}$.

   (b) $\lim_{n\to\infty}(\frac{1}{n+1}+\frac{1}{n+2}+\cdots\frac{1}{2n}) = \underline{(B)}$.

   (c) $\lim_{x\to\infty}\frac{1}{x\ln x}\int_1^x \ln t\,dt = \underline{(C)}$.

   (d) $\int \frac{1}{4x^2+4x+2}dx = \underline{(D)}$.

   (e) $\int_0^\infty e^{-\theta}\sin\theta\, d\theta = \underline{(E)}$.

   (f) Let $\Omega = \{(x,y) : x^2+y^2 \le 1\}$, then $\iint_\Omega |x|dA = \underline{(F)}$.

   (g) The radius of convergence for the power series expansion of

   $$\int_0^x \frac{\cos t - 1}{t}dt$$

   is $\underline{(G)}$.

   (h) The directional derivative for $f(x,y) = ye^x + xy^2$ at $(x,y) = (0,1)$ with respect to the vector $\mathbf{u} = (\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}})$ is $\underline{(H)}$.

計算與證明：請詳細寫出每一推導步驟。

2. Suppose $a_n > 0$, for all n and $\sum_{n=1}^\infty a_n$ converges, show that $\sum_{n=1}^\infty \left(\frac{a_n}{1+a_n}\right)$ converges. (10 points)

3. Prove that for $x, y \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $|\tan x - \tan y| \ge |x-y|$. (10 points)

4. Find the absolute maximum value of $f(x,y) = xy - x^3y^2$ over the square $0 \le x \le 1$, $0 \le y \le 1$. Give a complete discussion. (16 points)