

國立清華大學 106 學年度碩士班考試入學試題

系所班組別：科技管理研究所(0544)

考試科目（代碼）：微積分(4401)

共 頁，第 頁 *請在【答案卷】作答

1. (12 pts) Find an equation of the tangent to the curve $y^3 = xy + x + 1$ at the point $(0, 1)$.

2. (12 pts) Evaluate

$$\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x (1 - \tan 2t)^{1/t} dt.$$

3. (12 pts) A function f is defined by

$$f(x) = \int_0^\pi \sin t \sin(x - t) dt, \quad 0 \leq x \leq 2\pi.$$

Find the maximum value of f .

4. (12 pts) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{\log x}{\sqrt{xy}}$$

that satisfies the initial condition $y(1) = 0$.

5. (12 pts) If f is concave upward on $[a, b]$, show that

$$\frac{1}{b-a} \int_a^b f(x) dx > f\left(\frac{a+b}{2}\right).$$

6. (12 pts) If $a_0 + a_1 + a_2 + \cdots + a_k = 0$, show that

$$\lim_{n \rightarrow \infty} (a_0 \sqrt{n} + a_1 \sqrt{n+1} + a_2 \sqrt{n+2} + \cdots + a_k \sqrt{n+k}) = 0.$$

7. (14 pts) Find the minimum distance from the origin in \mathbb{R}^3 to the surface $z = 6xy + 7$.

8. (14 pts) Evaluate

$$\int \int_R \frac{1}{(x^2 + y^2)^2} dx dy,$$

where R is the plane region determined by the conditions $x^2 + y^2 \leq 1$ and $x + y \geq 1$.