

國立清華大學 105 學年度碩士班考試入學試題

系所班組別：科技管理研究所(0542)/乙組

考試科目 (4201)：微積分

共 1 頁，第 1 頁 *請在【答案卷、卡】作答

- (8 pts) Suppose that f and g are differentiable functions such that $f(g(x)) = x^2$ and $f'(x) = 1 + [f(x)]^3$. Find $g'(2)$.
- (24 pts) Find the limit, if it exists, or show that the limit does not exist.

(a) $\lim_{x \rightarrow \infty} (\sqrt{x+3\sqrt{x}} - \sqrt{x})$
(b) $\lim_{n \rightarrow \infty} \frac{1}{n} (\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{n\pi}{n})$
(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^4 + y^2}$

- (16 pts)

(a) Evaluate $\int_0^{\infty} e^{-x^2} dx$.
(b) Evaluate $\int_0^{\infty} \frac{e^{-x} - e^{-2x}}{x} dx$.

- (12 pts) Solve the initial value problem

$$x \frac{dy}{dx} - y = x^2 \log x, \quad y(1) = 0.$$

- (12 pts) Suppose f is differentiable in a neighborhood of c and $f'(c) > 0$. Show that there is a $\delta > 0$ such that $f(a) < f(c) < f(b)$ for all a in $(c - \delta, c)$ and all b in $(c, c + \delta)$.
- (12 pts) Let $T(x, y)$ be the temperature at the point (x, y) on the ellipse

$$x = 2\sqrt{2} \cos t, \quad y = \sqrt{2} \sin t, \quad 0 \leq t \leq 2\pi,$$

and suppose that $\partial T / \partial x = y$, $\partial T / \partial y = x$. Locate the maximum and minimum temperatures on the ellipse by examining dT/dt and d^2T/dt^2 .

- (16 pts)

(a) Expand $f(x) = \frac{x}{(1-x)^2}$ as a power series.
(b) Use part (a) to find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{3^n}$.

請務必書寫演算過程，否則不予計分