國立清華大學 105 學年度碩士班考試入學試題

系所班組別：科技管理研究所(0541)/甲組

考試科目（代碼）：統計學(4102)

　　*請在【答案卷、卡】作答

請依序作答，否則後果自行負責。只需寫下答案與簡短解釋，計算過程不需附上。

1. (10%) Mary tosses n+1 coins and John tosses n coins. What is the probability that Mary gets more heads than John?

2. (10%) The department photocopier has three parts A, B, C which can go wrong. The probability that A will fail during a copying session is 10^{-5}. The probability that B will fail is 10^{-1} if A fails and 10^{-5} otherwise. The probability that C will fail is 10^{-1} if B fails and 10^{-5} otherwise. The "Call Engineer" sign lights up if two or three parts fail. If only two parts have failed I can repair the machine myself but if all three parts have failed my attempts will only make matters worse. What is the probability of 3 parts fail conditional on 2 or 3 part fail? (The probability is close to which one? 1/5, 1/10, 1/20 1/30, 1/40, 1/50, 1/60 or 1/100)

3. (10%) Let X and Y be independent and normally distributed random variables with the same PDF

$$\frac{1}{\sqrt{2\pi}}e^{-x^2/2}.$$

Find the PDFs of X^2. (Please write down the formula of the PDF)

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4. (10%) Students of mathematics in a large university are given a percentage mark in their annual examination. In a sample of nine students the following marks were found:

28, 32, 34, 39, 41, 42, 42, 46, 56.

Students of history also receive a percentage mark. A sample of five students reveals the following marks:

53, 58, 60, 61, 68.

Does these data support the hypothesis that the marks for mathematics are more variable than the marks for history? Quantify your conclusion. Comment on your modelling assumption.

Additional Information:

Distribution:	N(0,1)	F(9,5)	F(8, 4)	$\chi^2(14)$	$\chi^2(13)$	$\chi^2(12)$
95% percentile	1.65	4.78	6.04	23.7	22.4	21.0

5. (10%) The independent observations X_1, X_2 are distributed as Poisson random variables with means μ_1, μ_2 respectively, where

$\ln \mu_1 = \alpha;$

$\ln \mu_2 = \alpha + \beta,$

with α, β unknown parameters. Find the maximum likelihood estimators of α.

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6. (10%) Suppose that $Z_i \sim N(0,1)$ and Z_i are independent. Please find the limiting distribution of

$$\sum_{i=1}^{n}\left(Z_i + \frac{1}{n}\right) / \sqrt{n}.$$

7. (10%) The assumptions of single-factor ANOVA can be described succinctly by means of the model equation $X_{i,j} = \mu_i + \varepsilon_{i,j}$ where $\varepsilon_{i,j}$ represents a random deviation from the population or true treatment mean μ_i. Which of the following statements are not true?

   (A) The $\varepsilon_{i,j}$'s are assumed to be independent.

   (B) The $\varepsilon_{i,j}$'s are normally distributed random variables.

   (C) $E(\varepsilon_{i,j}) = 0$ for every i and j.

   (D) $Var(\varepsilon_{i,j}) = \sigma^2$ for every i and j.

   (E) None of the above statements are true.

8. (10%) In the fixed effects model with interaction, assume that there are 5 levels of factor A, 4 levels of factor B, and 3 observations (replications) for each of the 20 combinations of levels of the two factors. Then, what is the number of degrees of freedom of the interaction sum of squares (SSAB)?

   (A) 60 (B) 20 (C) 15 (D) 32 (E) 12

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9. (10%) Answer the following questions with "True" or "False" and explain your answer very briefly.
   (A) Given the model $Y_t = X_t\beta + \varepsilon_t$, suppose that ε_t is NOT normally distributed. In this case, the maximum likelihood estimator (MLE) can not be derived.
   (B) Given the model $Y_t = X_t\beta + \varepsilon_t$, the ordinary least squares (OLS) of β is always equivalent to the MLE estimator of β.

10. (10%) Answer the following questions with "True" or "False" and explain your answer very briefly.
   (C) Given an estimated regression model, if the *F*-test for all the coefficients (except the constant term) equal to zero is significant at the level α, then all the individual *t*-ratios must also be significant at the same level.
   (D) In a linear model where ε_t are $N(0,\sigma^2)$. Then the residual $\hat{\varepsilon}_t$ is also distributed as $N(0,\sigma^2)$.

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Standard normal cumulative distribution function $\Phi(z)$ and $100 \times \gamma$th percentiles z_γ

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8314	0.3340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

γ	0.90	0.95	0.975	0.99	0.995	0.999	0.9995	0.99995	0.999995
z_γ	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417

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100 × γth Percentiles $t_\gamma(v)$ of Student's *t* distribution with *v* degrees of freedom

$$\gamma = \int_{-\infty}^{t_\gamma(v)} f(t;\, v)\, dt$$

	γ								
v	**0.60**	**0.70**	**0.80**	**0.90**	**0.95**	**0.975**	**0.99**	**0.995**	**0.9995**
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657	636.619
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	31.598
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	12.924
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.587
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.437
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	4.318
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012	4.221
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977	4.140
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	4.073
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921	4.015
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.965
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.922
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.883
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.850
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.819
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.792
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.767
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.745
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.725
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.707
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.690
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.674
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.659
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.646
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.551
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.460
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.291

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100 × γth Percentiles $\chi^2_\gamma(v)$ of the chi-square distribution with v degrees of freedom

$$\gamma = \int_0^{\chi^2_\gamma(v)} h(y; v)\, dy$$

v	0.005	0.010	0.025	0.050	0.100	0.250	0.500	0.750	0.900	0.950	0.975	0.990	0.995	0.999
1					0.02	0.10	0.45	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	0.01	0.02	0.05	0.10	0.21	0.58	1.39	2.77	4.61	5.99	7.38	9.21	10.60	13.82
3	0.07	0.11	0.22	0.35	0.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	0.21	0.30	0.48	0.71	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	0.41	0.55	0.83	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	0.68	0.87	1.24	1.64	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	0.99	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.96	26.12
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	4.60	5.23	6.26	7.26	8.55	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	5.14	5.81	6.91	7.96	9.31	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	5.70	6.41	7.56	8.67	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.73	40.79
18	6.26	7.01	8.23	9.39	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	6.84	7.63	8.91	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	35.58	43.82
20	7.43	8.26	9.59	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	8.03	8.90	10.28	11.59	13.24	16.34	20.34	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	8.64	9.54	10.98	12.34	14.04	17.24	21.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	9.26	10.20	11.69	13.09	14.85	18.14	22.34	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	9.89	10.86	12.40	13.85	15.66	19.04	23.34	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	16.47	19.94	24.34	29.34	34.38	37.65	40.65	44.31	46.93	52.62
30	13.79	14.95	16.79	18.49	20.60	24.48	29.34	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	29.05	33.66	39.34	45.62	51.80	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	37.69	42.94	49.33	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	46.46	52.29	59.33	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	55.33	61.70	69.33	77.58	85.53	90.53	95.02	100.42	104.22	112.32
80	51.17	53.54	57.15	60.39	64.28	71.14	79.33	88.13	96.58	101.88	106.63	112.33	116.32	124.84
90	59.20	61.75	65.65	69.13	73.29	80.62	89.33	98.64	107.56	113.14	118.14	124.12	128.30	137.21
100	67.33	70.06	74.22	77.93	82.36	90.13	99.33	109.14	118.50	124.34	129.56	135.81	140.17	149.45

or large v, $\chi^2_\gamma(v) \doteq v[1 - (2/9v) + z_\gamma\sqrt{(2/9v)}]^3$.