

國立清華大學命題紙

乙組(財務金融組)

99學年度 計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 1 頁，第 1 頁 \*請在【答案卷卡】作答  
4603

1. Consider a random variable  $X$  with a Poisson distribution as follows.

$$P(X) = \frac{e^{-\lambda} \lambda^X}{X!}$$

where  $\lambda$  is the expect number of events and  $X = 0, 1, 2, \dots, \infty$ . Compute  $E(X^4)$ . (10%)

2. The random variables  $X$  and  $Y$  with the joint probabilities are given as follows:

X \ Y	-1	0	1	$f_X(X)$
1	0.2	0	0.1	0.3
2	0.1	0.2	0.2	0.5
3	0.1	0.1	0	0.2
$f_Y(Y)$	0.4	0.3	0.3	1

a. Suppose  $Z = Y^2$ , compute the correlation coefficient of  $X$  and  $Z$ . (5%)

b. Compute the expectation and variance of  $X$  when  $Y=0$ . (5%)

c. Suppose  $W = \alpha + \beta X + \gamma X^2$ , compute  $\text{Var}(W)$ . (10%)

3. You want to have 95% confidence of estimating the proportion of office workers who respond to e-mail within an hour to within  $\pm 0.05$ . Because you have not previously undertaken such a study, there is no information available from past data. Determine the sample size needed? (10%)

4. Microsoft Excel was used on a set of data involving the number of parasites found on 46 Monarch butterflies captured in Pismo Beach State Park. A biologist wants to know if the mean number of parasites per butterfly is over 20. She will make her decision using a test with a level of significance of 0.10. The following information was extracted from the Microsoft Excel output for the sample of 46 Monarch butterflies:

$n = 46$ ; Arithmetic Mean = 28.00; Standard Deviation = 25.92; Standard Error = 3.82;

Null Hypothesis:  $H_0: \mu \leq 20.000$ ;  $\alpha = 0.10$ ;  $df = 45$ ;  $t$ -Test Statistic = 2.09;

One-Tailed Test Upper Critical Value = 1.3006;  $p$ -value = 0.021; Decision = Reject.

- What is the power of the test if the mean number of parasites per butterfly on Monarch butterflies in Pismo Beach State Park is 18 using a 5% level of significance and assuming that the population standard deviation is 25.92. (10%)

國立清華大學命題紙

乙組(財務金融組)

99學年度計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 7 頁，第 2 頁 \*請在【答案卷卡】作答  
4603

5. At a computer manufacturing company, the actual size of computer chips is normally distributed with a mean of 1 centimeter and a standard deviation of 0.1 centimeter. A random sample of 12 computer chips is taken. Above what value do 2.5% of the sample means fall? (10%)

6. A hotel chain has identically sized resorts in 5 locations. The data that follow resulted from analyzing the hotel occupancies on randomly selected days in the 5 locations.

<u>ROW</u>	<u>Caymen</u>	<u>Pennkamp</u>	<u>California</u>	<u>Mayaguez</u>	<u>Maui</u>
1	28	40	21	37	22
2	33	35	21	47	19
3	41	33	27	45	25

Analysis of Variance

Source	<i>df</i>	SS
Location	4	963.6
Error	10	210.0
Total		

a. Assuming that the population variances from 5 locations are equal, at 5% level of significance, test the null hypothesis that all five locations have equal population means. (5%)

b. Test the assumption that the population variances from 5 locations are equal at 5% significance level? (10%)

7. One criterion used to evaluate employees in the assembly section of a large factory is the number of defective pieces per 1,000 parts produced. The quality control department wants to find out whether there is a relationship between years of experience and defect rate. Since the job is repetitious, after the initial training period any improvement due to a learning effect might be offset by a loss of motivation. A defect rate is calculated for each worker in a yearly evaluation. The results for 100 workers are given in the table below.

		Years Since Training Period		
		< 1 Year	1 - 4 Years	5 - 9 Years
Defect Rate:	High	6	9	9
	Average	9	19	23
	Low	7	8	10

At 5% level of significance, is there evidence of a relationship between defect rate and years of experience? (10%)

國立清華大學命題紙

乙組(財務金融組)

99學年度 計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 7 頁，第 3 頁 \*請在【答案卷卡】作答  
4603

8. Let  $X_1, X_2, \dots, X_n$  be a sample of size n from a normal distribution  $N(\mu, \sigma^2)$ . Consider the following point estimators of  $\sigma^2$  :

$$S_1^2 = \frac{\sum_{i=1}^{i=n} (X_i - \bar{X})^2}{n-1}, \quad S_2^2 = \frac{\sum_{i=1}^{i=n} (X_i - \bar{X})^2}{n}, \quad S_3^2 = \frac{n}{n-1} (X_1 - \bar{X})^2$$

- Which of these estimators are unbiased? (10%)
- Which of these estimators are consistent? (5%)

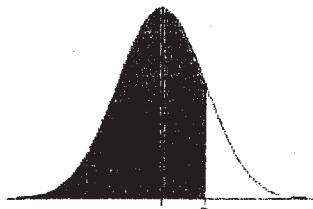
國立清華大學命題紙

## 乙組(財務工程組)

99 學年度 計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 7 頁，第 4 頁 \*請在【答案卷卡】作答  
4603

## Standard Normal Cumulative Probability Table



**Cumulative probabilities for POSITIVE z-values are shown in the following table:**

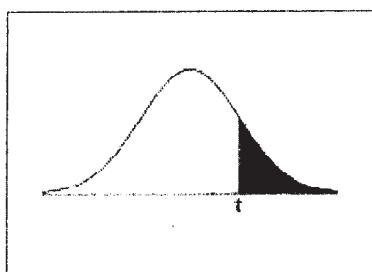
國立清華大學命題紙

乙組(財務金融組)

99學年度 計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 7 頁，第 5 頁 \*請在【答案卷卡】作答  
4603

t-Distribution Table



The shaded area is equal to  $\alpha$  for  $t = t_\alpha$ .

$df$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.434	2.719
38	1.304	1.686	2.024	2.429	2.712
$\infty$	1.282	1.645	1.960	2.326	2.576

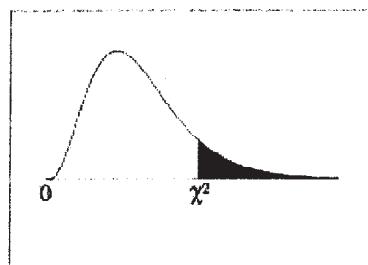
國立清華大學命題紙

乙組(財務金融組)

99學年度 計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 7 頁，第 6 頁 \*請在【答案卷卡】作答  
4603

Chi-Square Distribution Table



The shaded area is equal to  $\alpha$  for  $\chi^2 = \chi^2_\alpha$ .

$df$	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

國立清華大學命題紙

乙組(財務金融組)

99學年度 計量財務金融學系甲組(財務金融組) 碩士班入學考試

科目 統計學 科目代碼 4503 共 7 頁，第 7 頁 \*請在【答案卷卡】作答  
4603

Table A.6 F-Distribution, Upper 5% Points ( $F_{0.05}$ )<sup>a</sup>

	Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	$\infty$
Degrees of Freedom																			
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	1.98	1.93	1.88	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.75
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.66	1.61	1.55	1.50	1.43	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

<sup>a</sup>Interpolation should be performed using reciprocals of the degrees of freedom.

Source: Reproduced from M. Merrington, C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (F) Distribution," Biometrika, Vol. 33, 1943, p. 73, with the permission of the Biometrika Trustees.