國　立　清　華　大　學　命　題　紙

96 學年度　計量財務金融學　系（所）　甲　組碩士班入學考試

科目　統計學　科目代碼　5203 共 4 頁第 1 頁 *請在【答案卷卡】內作答

Please explain all of your answers except for question 1, 2 and 3. There will be no credit if you simply write down the final answer.

1. (5%) X is a continuous random variable following uniform distribution between 3 and 10. We can write it as X~uniform(3, 10). Please calculate the probability Pr(X=4)

2. (5%) If we build a regression model to fit the weights of elementary school students with age and height, and derive the regression line as follows: weight = 10 + 0.2 x age + 0.02 x height
Which explanatory variable seems to be more related to weights? (A) age (B) height (C) cannot tell.

3. (5%) We can approximate the distribution of a sample proportion with normal distribution when the sample number is large. This is based on (A) Law of large number (B) Bayes' Law (C) Central Limit Theorem (D) maximum likelihood

4. (15%) To know whether waitresses earn larger tips than waiters, we did a survey with 50 randomly selected restaurants. We record the tips for one randomly selected waiter and one randomly selected waitress in each of the restaurant for a one-week period. The data can be summarized as follows. If $x$ represents data from 50 waiters and $y$ represents data from 50 waitresses, please answer our original question with significance level 0.05. You need to state clearly your hypotheses and specify the parameters of interests.

$$\sum_{i=1}^{50} x_i = 50067, \sum_{i=1}^{50} y_i = 50263, \sum_{i=1}^{50} x_i^2 = 50146957, \sum_{i=1}^{50} y_i^2 = 50540081,$$

$$\sum_{i=1}^{50} x_i y_i = 50343049, \bar{x} = 1001.34, \bar{y} = 1005.26,$$

$$\sum_{i=1}^{50} (x_i - \bar{x})^2 = 12867.22, \sum_{i=1}^{50} (y_i - \bar{y})^2 = 12697.62, \sum_{i=1}^{50} (x_i - \bar{x})(y_i - \bar{y}) = 12696.58$$

5. (10%) There are three candidates, X, Y and Z for the coming election. During a survey from mass media, 53%, 19.8%, 27.2% of the sampled voters will vote for X, Y, and Z respectively. In general, 80%, 30%, 25% of supporters for party A, B and C will vote candidate X, while 10%, 60% and 22% of supporters for party A, B and C will vote candidate Y. If the sampled voters can all be grouped into the supporters for any one of the three parties, what is the proportion of supporters for each of the three parties in this survey?

6. (a) (10%) We want to design an opinion poll to estimate the public support for party A. If we want the 95% confidence interval of the rate to be within 1% of the sample proportion (sample proportion ± 1%), what is the minimum number of samples needed for this estimation?

   (b) (10%) Suppose the public support for party A is 0.64 based on the record of last year. If we conduct an opinion poll with 900 samples this year and derive the supporting rate for party A as 0.6, please test if the public support for party A has significantly decreased since last year. Use significance level 0.05 for your test.

7. (7%) If the variances of two random variables X and Y are both $\sigma^2$, what is the maximum possible variance of their average?

8. A teacher randomly sampled 11 students from each of his four classes and compared the scores with the following ANOVA table. Please fill in the cell (a) to (d).

| Source | DF | SS | MS | F | p-value |
|---|---|---|---|---|---|
| Class | (a) (3%) | 1154 | | (c) (6%) | (d) (6%) |
| Residual | | 4379 | | | |
| Total | (b) (3%) | | | | |

   (e) (10%) If we compare the grades of class 1 ($x_i$) and 2 ($y_i$) with the following numbers, please conclude if the there is any difference between the two classes with significance level 0.05. Simply assume equal variance here.

$$\sum_{i=1}^{11} x_i = 630, \sum_{i=1}^{11} y_i = 592, \sum_{i=1}^{11} (x_i - \bar{x})^2 = 998.2, \sum_{i=1}^{11} (y_i - \bar{y})^2 = 779.6$$

   (f) (5%) Is there any conflict of the conclusions derived from (d) and (e)? Why or why not?

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科目　統計學　科目代碼　5203 共 4 頁第 3 頁 *請在【答案卷卡】內作答

Some numbers you might need:

| | F(1,44) | F(1,43) | F(1,41) | F(1,40) | F(2,44) | F(2,43) | F(2,41) | F(2,40) |
|---|---|---|---|---|---|---|---|---|
| 99% quantile | 7.248 | 7.264 | 7.296 | 7.314 | 5.123 | 5.136 | 5.163 | 5.179 |
| 97.5% quantile | 5.386 | 5.395 | 5.414 | 5.424 | 4.016 | 4.024 | 4.042 | 4.051 |
| 95% quantile | 4.062 | 4.067 | 4.079 | 4.085 | 3.209 | 3.214 | 3.226 | 3.232 |
| 90% quantile | 2.823 | 2.826 | 2.832 | 2.835 | 2.427 | 2.430 | 2.437 | 2.440 |

| | F(3,44) | F(3,43) | F(3,41) | F(3,40) | F(4,44) | F(4,43) | F(4,41) | F(4,40) |
|---|---|---|---|---|---|---|---|---|
| 99% quantile | 4.261 | 4.273 | 4.299 | 4.313 | 3.778 | 3.790 | 3.815 | 3.828 |
| 97.5% quantile | 3.430 | 3.438 | 3.454 | 3.463 | 3.093 | 3.101 | 3.117 | 3.126 |
| 95% quantile | 2.816 | 2.822 | 2.833 | 2.839 | 2.584 | 2.589 | 2.600 | 2.606 |
| 90% quantile | 2.213 | 2.216 | 2.222 | 2.226 | 2.077 | 2.080 | 2.087 | 2.091 |

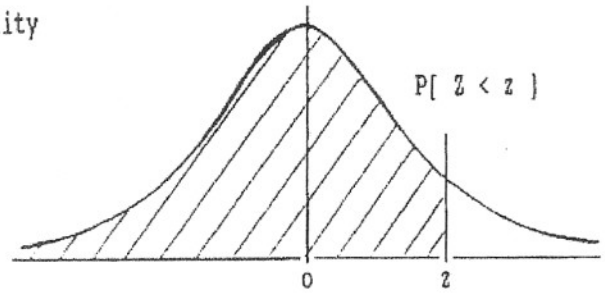
$t_{0.975,22}=2.073$, $t_{0.95,22}=1.717$, $t_{0.9,22}=1.32$, $t_{0.975,21}=2.080$, $t_{0.95,21}=1.721$, $t_{0.9,21}=1.323$ $t_{0.975,20}=2.086$, $t_{0.95,20}=1.725$, $t_{0.9,20}=1.325$, $t_{0.95,99}=1.660$, $t_{0.95,98}=1.661$, $t_{0.95,49}=1.676$, $t_{0.95,48}=1.677$,

# STANDARD STATISTICAL TABLES

## 1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\,Z < z\,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5159 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7854 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8804 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9773 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9865 | 0.9868 | 0.9871 | 0.9874 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9924 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9980 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| z | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
| P | 0.9986 | 0.9990 | 0.9993 | 0.9995 | 0.9997 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 1.0000 |