

注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。


國立清華大學 109 學年度碩士班考試入學試題

系所班組別：計量財務金融學系
乙組(財務工程組)

科目代碼：5003

考試科目：微積分

—作答注意事項—

1. 請核對答案卷(卡)上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清(含未依範例畫記)致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

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系所班組別：計量財務金融學系碩士班 乙組

考試科目 (代碼)：微積分 (5003)

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Part I: Multiple choice (21%, 3 points each. one answer for each question)

- $\lim_{x \rightarrow 0} \frac{1 - \exp(x^{2020})}{x \log(1 + x^{2019})} = ?$
(A) -2 (B) -1 (C) 0 (D) 1
- $\sum_{n=0}^{\infty} \frac{3n \exp(n)}{n^2 + 1} = ?$
(A) $\sqrt{\pi}$ (B) π (C) ∞ (D) $\exp(1)$
- Let $f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$. Which of the following statement is true?
(A) f is continuous at $(0, 0)$. (B) f is differentiable at $(0, 0)$.
(C) $D_u f(0, 0) = \frac{3}{2}$ for $u = (\frac{\sqrt{3}}{2}, \frac{1}{2})$ (D) $f_x(0, 0) = f_y(0, 0) = 1$
- The total number of critical points in $f(x, y) = (x^2 + y^2)e^{-x}$ equals
(A) 0 (B) 1 (C) 2 (D) more than 2.
- Find $\lim_{x \rightarrow \infty} (1 - \frac{1}{x} - \frac{2}{x})^x =$
(A) 1 (B) e^{-1} (C) e^{-2} (D) e^{-3}
- Let $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$. Which of the following is true?
(A) f is differentiable on \mathbb{R} . (B) $f(0)$ is a local minimum.
(C) $f(2)$ is a local maximum. (D) $(-1, -6)$ is an inflection point of $y = f(x)$.
- Let $f(x, y) = x^2 + kxy + 4y^2$. For what value of k will f have a local minimum at $(0, 0)$?
(A) 3 (B) 5 (C) 7 (D) 9

Part II: Fill in blanks (35%, 5 points each)

- Write a Taylor Series about $x = 0$ for this integral $\int_0^x \frac{\exp(t)-1}{t} dt =$ _____.
- If f is differentiable at a , where $a > 0$, then $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}} =$ _____.
- Consider the function $f(x, y, z) = xy + z^2$ on the unit ball $x^2 + y^2 + z^2 \leq 1$. The absolute maximum value is _____ (A) _____ and the absolute minimum value is _____ (B) _____.
- If $z = y + f(x^2 - y^2)$, where f is differentiable, then $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} =$ _____.
- For the function $T(x, y) = 20 - 4x^2 - y^2$, in which direction from $(2, -3)$ does T increase most rapidly?
_____ (A) _____ What is the rate of increase? _____ (B) _____

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考試科目（代碼）：微積分（5003）

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Part III: Show Your Calculation

- (14%) The optimal investment strategy $X \in \mathbb{R}^{n \times 1}$ concerns minimizing the risk, denoted by $\frac{1}{2} X^T \Sigma X$, in which Σ is an invertible $n \times n$ matrix and T denotes the transpose. Under the following two constraints:
 - fixed expected return $\mu^T X = m$, in which $\mu \in \mathbb{R}^{n \times 1}$ and $m \in \mathbb{R}$ are given,
 - the total sum of weights $\sum_{i=1}^n X_i = 1$,find out the optimal investment strategy $X^* \in \mathbb{R}^{n \times 1}$.
- (15%) Let $f(x) = \int_1^{2x} \sqrt{16+t^4} dt$.
 - Explain why the inverse function of f , denoted by f^{-1} , exist.
 - Solve for $(f^{-1})'(0)$.
- (15%) For a positive parameter λ , a real-valued function is defined by $f(x; \lambda) = \lambda \exp(-\lambda x)$; for $x \geq 0$, and $f(x) = 0$ otherwise.
 - When $\theta < \lambda$, calculate $M(\theta) = \int_{-\infty}^{\infty} e^{\theta x} f(x; \lambda) dx$.
 - Calculate $I(x) = \sup_{\theta \in \mathbb{R}} [\theta x - \ln M(\theta)]$.