國立清華大學 107 學年度碩士班考試入學試題

系所班組別：計量財務金融學系碩士班 乙組(財務工程組)

考試科目（代碼）：微積分（5004）

 *請在【答案卷、卡】作答

Part I Fill-in blanks (Total 40 points, 4 points each.)

(1) $\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n-1}}{n\sqrt{n}} =$ ____________

(2) Let $f(x) = \frac{1}{4}x^3 + x - 1$. $(f^{-1})'(-3) =$ ____________

(3) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\mathrm{d}x =$ ____________

(4) The interval of convergence of $\sum_{k=1}^{\infty} \frac{k}{5^k}x^k =$ ____________

(5) The total derivative of $f(x,y) = \ln(x^2+y^2) + x\tan^{-1} y$ is ____________

(6) Given $f(x,y) = x^2 - y^3 - 10x + 12y + 19$, find its saddlepoint ____________ and its extremum ____________

(7) $\int_0^{\infty} e^{-3x^2}\mathrm{d}x =$ ____________

(8) If $\frac{\mathrm{d}y}{\mathrm{d}x} + y = 2xe^{-x}$ and $y(0) = 1$, $y(x) =$ ____________

(9) The arc length of $r = 1 + \sin\theta$, $0 \le \theta \le 2\pi$ is ____________

Part II Calculation and Proof (Total 60 points, 15 points each.)

1. Given a real-valued function $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$ where m and σ are constants,
   (a) Calculate $M(\theta) = \int_{-\infty}^{\infty} e^{\theta x} f(x) \mathrm{d}x$. (Hint: $\int_{-\infty}^{\infty} f(x) \mathrm{d}x = 1$)
   (b) Define
   $$I(x) = \sup_{\theta} \left[\theta x - \ln M(\theta)\right].$$
   Compute $I(x)$ and find its minimum.

2. Given $\varphi_n(u) = \left(\frac{1}{2} e^{\frac{u}{\sqrt{n}}} + \frac{1}{2} e^{-\frac{u}{\sqrt{n}}}\right)^n$ and $\varphi(u) = e^{\frac{1}{2}u^2}$ with $u \in \mathbb{R}$ and $n > 0$, prove that for any $u \in \mathbb{R}$,
   $$\lim_{n \to \infty} \varphi_n(u) = \varphi(u).$$

3. Given proper constants of μ_i and σ_{ij} for $i, j \in \{1, 2\}$, minimize
   $$\frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} w_i w_j \sigma_{ij}$$
   over w_1 and w_2 with restriction to $\sum_{i=1}^{2} w_i \mu_i = \mu_p$ and $\sum_{i=1}^{2} w_i = 1$.

4. (a) State the fundamental theorem of calculus.
   (b) If $g(x)$, $h(x)$ and $k(x)$ are differentiable, differentiate
   $$F(x) = \int_{h(x)}^{k(x)} g(t) \mathrm{d}t$$