國立清華大學 105 學年度碩士班考試入學試題

系所班組別：計量財務金融學系碩士班 乙組

考試科目（代碼）：微積分 (4404)

 *請在【答案卷、卡】作答

1. (7 pts) Let f be twice differentiable, and let $g(x) = f(x^2 - 1)$. Suppose $f'(0) = 3$ and $f''(0) = 2$. Find $g''(1)$.

2. (12 pts) For what values of x does the series
$$\sum_{n=2}^{\infty} \frac{(1-x)^n}{n(\log n)^2}$$
converge?

3. (12 pts) Find the maximum and minimum values of
$$f(x,y) = 2x^2 + y^2 - y$$
on the unit disc $x^2 + y^2 \le 1$.

4. (12 pts) Sketch the region $R = \{(x,y) / 0 \le x \le y \text{ and } x^2 + y^2 \le 1\}$ and evaluate the integral
$$\int\int_R x\,dx\,dy.$$

5. (12 pts) Find the maximum value of
$$f(x,y,z) = 2^x 3^y 5^z$$
on the unit sphere $x^2 + y^2 + z^2 = 1$.

6. (15 pts)
   (a) Show that the function $f(x) = x^{1/x}$ is one-to-one on the interval $(0, e)$.
   (b) Compute $(f^{-1})'(\sqrt{2})$.

7. (15 pts) Let $a_1 = 2001$ and define a_n to be
$$a_n = \frac{3a_{n-1} + 1}{4} \qquad \text{for } n > 1.$$
Find the smallest positive integer n such that $a_n < \frac{4}{3}$.

8. (15 pts) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by
$$f(x,y) = \begin{cases} y \sin \frac{1}{x^2+y^2} & \text{if } (x,y) \ne (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$
   (a) Do $\partial f / \partial x$ and $\partial f / \partial y$ exist at the origin?
   (b) Is f continuous at the origin?

務必書寫計算過程，否則不予計分。