

# 國立清華大學 104 學年度碩士班考試入學試題

系所班組別：計量財務金融學系碩士班 甲組、乙組

考試科目（代碼）：統計學(4503)(4603)

共 9 頁，第 1 頁 \*請在【答案卷、卡】作答

請依序作答，否則後果自行負責。只需寫下答案與簡短解釋，計算過程不需附上。

1. (30%)

Consider a normally distributed random variable  $z_t$ :

$$z_t \sim N(0,1).$$

Denote the density function of  $z_t$  by  $f(z_t)$  and the implied probability measure by  $P$ . Define a Radon-Nikodym derivative

$$\eta(z_t) = \exp(z_t\alpha - 0.5\alpha^2).$$

When we multiply  $\eta(z_t)$  by  $P$ , we obtain a new probability measure  $Q$ . This can be seen from the following:

$$f(z_t)\eta(z_t) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(z_t - \alpha)^2\right\}.$$

Now assume that the return  $R_t$  of a stock has the following log-normal distribution:

$$\ln(R_t) \sim N(\mu, \sigma^2).$$

Suppose we let the density of  $\ln(R_t)$  be denoted by  $f(R_t)$ .

- (1) Find a function  $\eta(R_t)$  such that under the probability measure  $Q$ ,  $R_t$  has a mean equal to the risk-free rate  $r$ .
- (2) Find a  $\eta(R_t)$  such that under the probability measure  $Q$ ,  $R_t$  has a mean zero.
- (3) Is the variance different under these probabilities ( $P$  and  $Q$ )?

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2. (10%)

Assume that  $X_i \sim N(\mu, \sigma^2)$ ,  $i=1, \dots, 30$ , where  $\mu$  is known and  $\sigma^2$  has to be estimated. Try to find the shortest 98% confidence interval for  $\sigma^2$ .

3. (20%)

The independent observations  $x_1$  and  $x_2$  are distributed as Poisson random variables with mean  $\mu_1$  and  $\mu_2$  respectively, where

$$\ln \mu_1 = \alpha,$$

$$\ln \mu_2 = \alpha + \beta,$$

with  $\alpha$  and  $\beta$  unknown parameters.

(1) Find the maximum likelihood estimator of  $\alpha$ .

(2) Find the maximum likelihood estimator of  $\beta$ .

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4. (15%)

Continuous random variables  $X$  and  $Y$  has a joint PDF

$$f(x, y) = \frac{(m+n+2)!}{m!n!} (1-x)^m y^n,$$

for  $0 < y \leq x < 1$ , where  $m, n$  are given positive integers. Please calculate

$$P\left(Y \leq \frac{1}{3} \mid X = \frac{2}{3}\right) = ?$$

5. (10%)

Assume that if the stock price is  $S_0$  at the beginning of the period, it will be either  $u \times S_0$  or  $d \times S_0$  at the end of the period (with risk-neutral probabilities  $p$  and  $1-p$ , respectively), where the multiplicative factors  $u$  and  $d$  are constant and known. The value of a European derivatives is the discounted expectation of its value at maturity, discounting at the risk-free rate  $r$  under the risk-neutral probability.

(1) Consider a European put option which value at maturity is  $\max\{K - S_N, 0\}$ . Please find the value of a European put option in a one period model ( $N = 1$ ).

(2) Find the value of a European put option in a general  $N$  period model.

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6. (15%)

Grades for a random sample of students who have taken statistics from a certain professor over the past 25 year were used to estimate the relationship between

$y$ =grade on the final exam and

$x$ =average of home-works (Avg.H.W.)

The regression equation is

Final = 16.6 + 0.784 Avg.H.W.

Predictor	Coef	StDev	$t$
Constant	16.609	4.246	3.91
Avg.H.W	0.78357	0.05593	14.01

S = 9.801      R-Sq = 52.4%      R-Sq(adj) = 52.2%

Analysis of Variance

Source	DF	SS	MS	$F$
Regression	1	18855	18855	196.29
Residual Error	178	17097	96	
Total	179	35952		

Fit	StDev Fit	95.0% CI	95.0% PI
75.377	0.731	( 73.935, 76.818)	( 55.982, 94.771)

- (1) The results for a test of  $H_0:\beta_1 = 0$  versus  $H_a:\beta_1 \neq 0$  show that
- A. The null hypothesis can be rejected because  $F=196.29$  and  $p$ -value=0.000
  - B. The null hypothesis can be rejected because  $t=14.01$  and  $p$ -value=0.000
  - C. The null hypothesis can be rejected because  $F=196.29$  and  $p$ -value=0.01
  - D. The null hypothesis can be rejected because  $t=14.01$  and  $p$ -value=0.01

國立清華大學 104 學年度碩士班考試入學試題

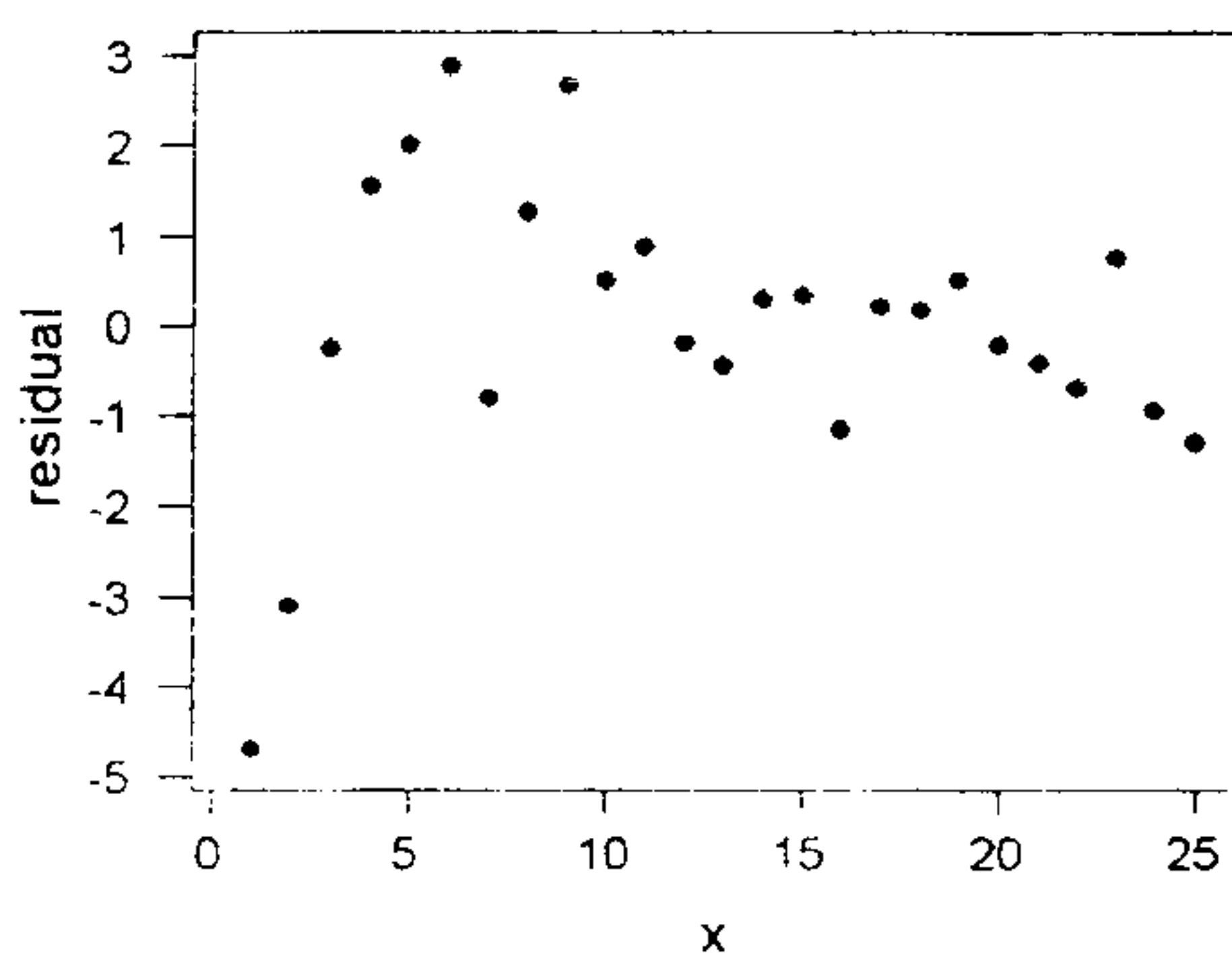
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- (2) What is the best way to determine whether or not there is a statistically significant linear relationship between two quantitative variables?
- A. Compute a regression line from a sample and see if the sample slope is 0.
  - B. Compute the correlation coefficient and see if it is greater than 0.5 or less than  $-0.5$ .
  - C. Conduct a test of the null hypothesis that the population slope is 0.
  - D. Conduct a test of the null hypothesis that the population intercept is 0.
- (3) A regression line is used for all of the following EXCEPT one. Which one is not a valid use of a regression line?
- A. to *estimate* the average value of  $y$  at a specified value of  $x$ .
  - B. to *predict* the value of  $y$  for an individual, given that individual's  $x$ -value.
  - C. to *estimate* the change in  $y$  for a one-unit change in  $x$ .
  - D. to determine if a change in  $x$  *causes* a change in  $y$ .
- (4) Consider the following plot of residuals versus  $x$  for a regression analysis.



Which statement is NOT true about the regression model?

- A. The sum of the residuals = 0.
- B. The independent variable ranges from 1 to 25.
- C. The residual plot may shows a nonrandom pattern of residuals.
- D. The residual plot shows a random pattern of residuals.

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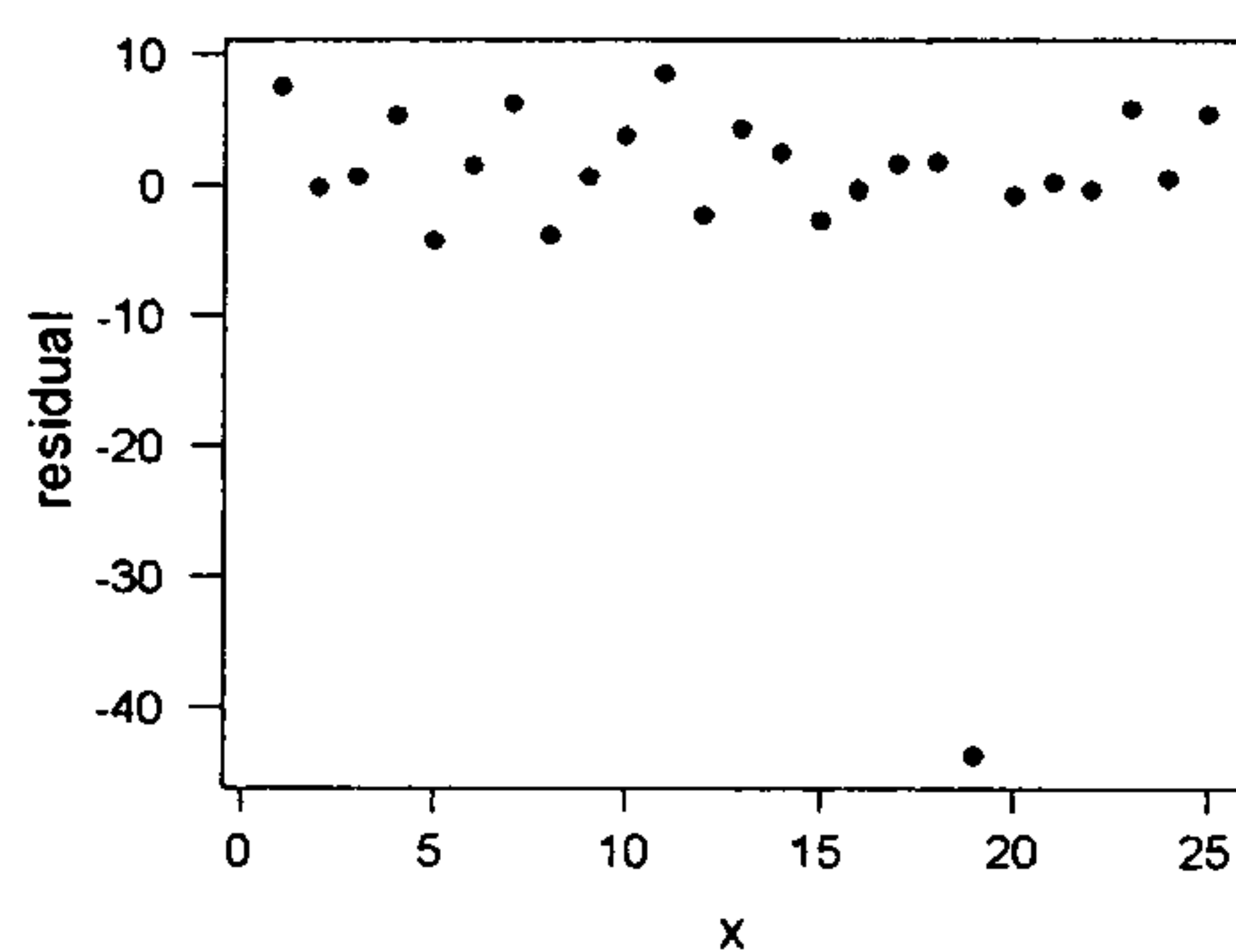
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- (5) Consider the following plot of residuals versus  $x$  for a regression analysis.



Based on the plot, what problem with the regression model or data is most noticeable?

- A. The variances are not constant.
- B. The mean of  $Y$  is not a linear function of  $X$ .
- C. The variance of  $Y$  is not constant at each  $X$ .
- D. There is an outlier in the data.

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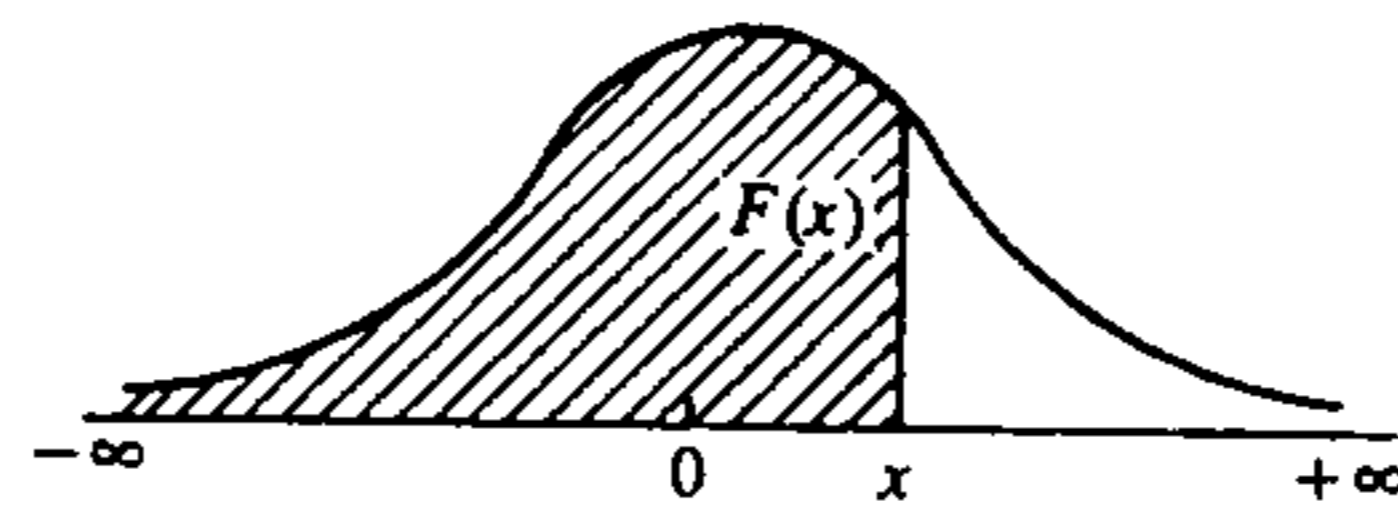
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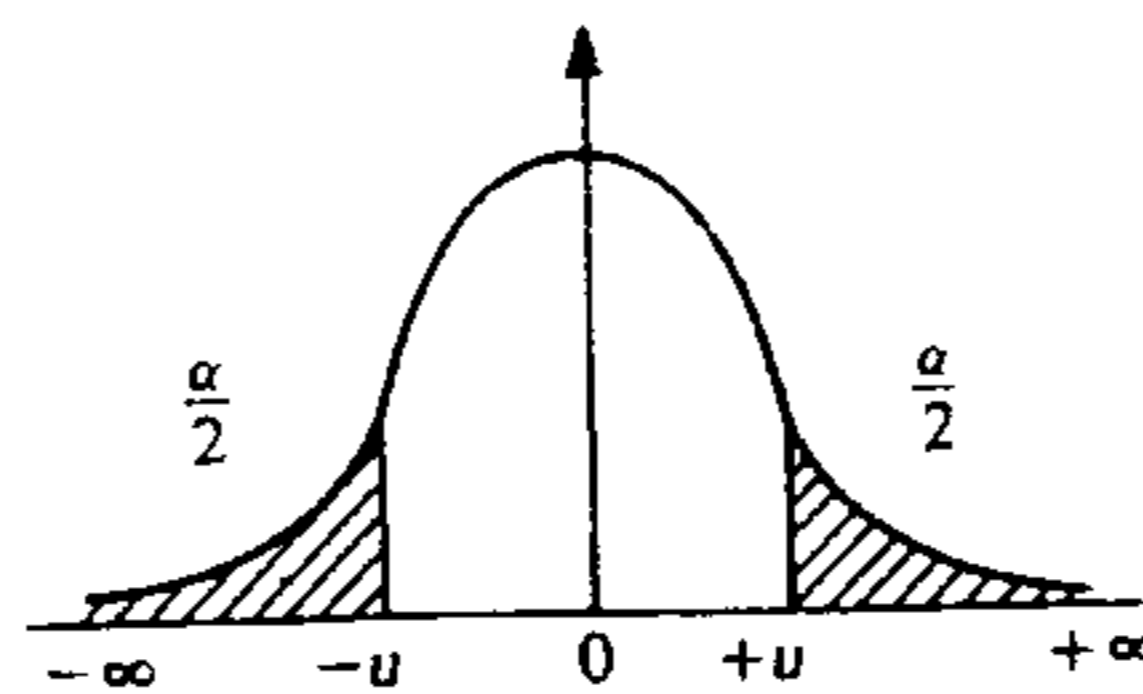
Table 1  
Cumulative Distribution Function of the Standard Normal Distribution



$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-u^2/2} du$$

$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56750	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59484	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67365	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76731	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80231	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84850	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92786	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95819	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861

Table 2  
Quantiles of the Standard Normal Distribution ( $u =$  value of  $Z$  such that  $Pr(|Z| > u) = \alpha$ )



$\alpha$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	$\infty$	2.5758	2.3263	2.1701	2.0537	1.9600	1.8808	1.8119	1.7507	1.6954
0.1	1.6449	1.5982	1.5548	1.5141	1.4758	1.4395	1.4051	1.3722	1.3408	1.3106
0.2	1.2816	1.2536	1.2566	1.2004	1.1750	1.1503	1.1264	1.1031	1.0803	1.0581
0.3	1.0364	1.0152	0.9945	0.9741	0.9542	0.9346	0.9154	0.8965	0.8779	0.8596
0.4	0.8416	0.8239	0.7892	0.8064	0.7722	0.7554	0.7388	0.7255	0.7063	0.6903
0.5	0.6745	0.6588	0.6433	0.6280	0.6128	0.5978	0.5828	0.5681	0.5534	0.5388
0.6	0.5244	0.5101	0.4959	0.4817	0.4677	0.4538	0.4399	0.4261	0.4125	0.3989
0.7	0.3853	0.3719	0.3585	0.3451	0.3319	0.3186	0.3000	0.2924	0.2793	0.2663
0.8	0.2533	0.2404	0.2275	0.2147	0.2019	0.1819	0.1764	0.1637	0.1510	0.1383
0.9	0.1257	0.1130	0.1004	0.0878	0.0753	0.0627	0.0502	0.0376	0.0251	0.0125

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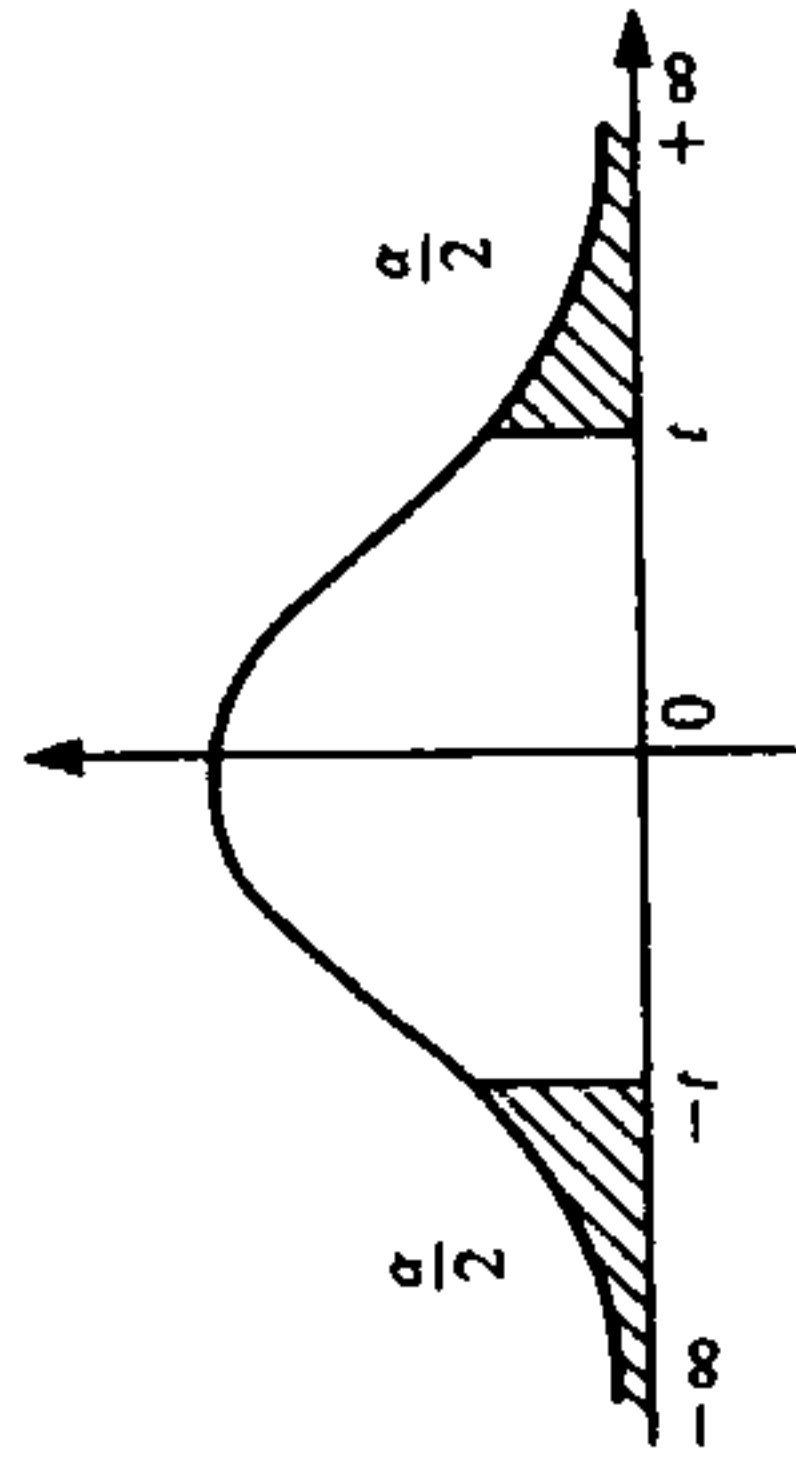
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共 9 頁，第 8 頁

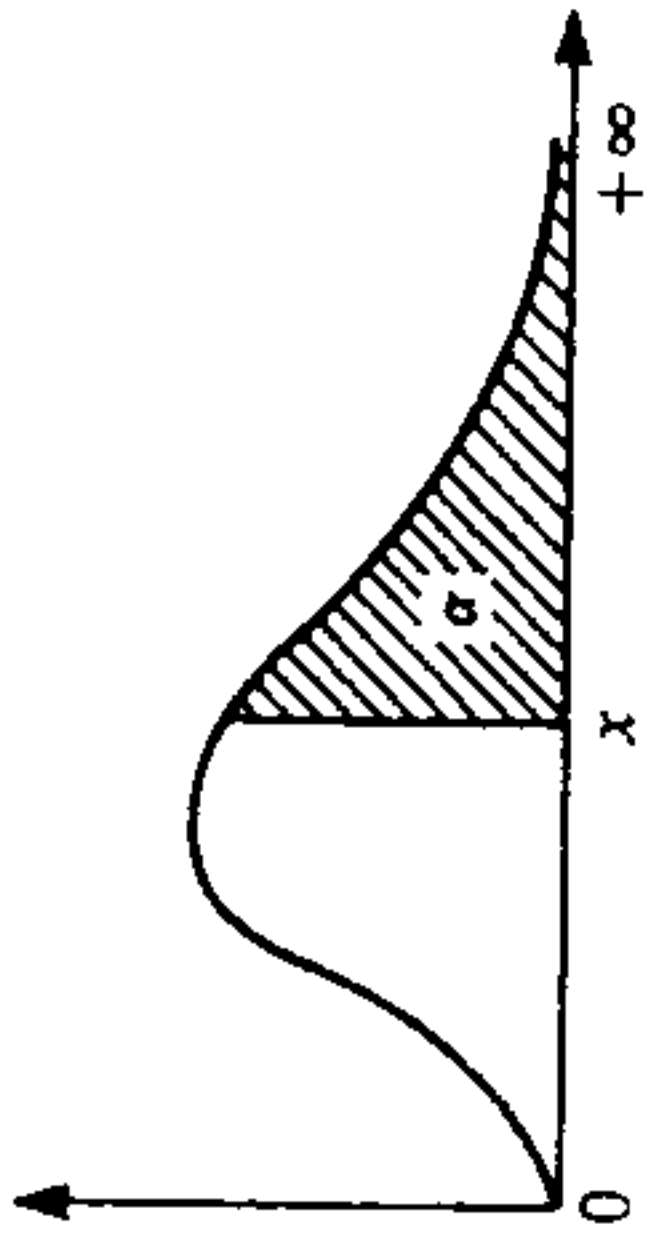
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Table 4  
Quantiles of the Student Distribution with  $\nu$  Degrees of Freedom  
( $t =$  value of  $T$  such that  $Pr(|T| > t) = \alpha$ )



$\alpha$	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.001
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.142	0.289	0.445	0.617	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.941	12.929
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.131	0.265	0.404	0.559	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.130	0.263	0.402	0.549	0.711	0.896	1.119	1.415	1.895	2.447	3.143	3.707	5.959
7	0.129	0.262	0.399	0.546	0.706	0.889	1.108	1.397	1.860	2.306	2.998	3.499	5.408
8	0.129	0.261	0.398	0.543	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	5.041
9	0.129	0.260	0.397	0.542	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.781
10	0.129	0.260	0.396	0.540	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
11	0.128	0.259	0.395	0.539	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
12	0.128	0.259	0.394	0.538	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
13	0.128	0.258	0.393	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
14	0.128	0.258	0.393	0.536	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
15	0.128	0.258	0.392	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
16	0.128	0.257	0.392	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
17	0.127	0.257	0.391	0.533	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
18	0.127	0.257	0.391	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
19	0.127	0.257	0.391	0.532	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
20	0.127	0.256	0.390	0.532	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
21	0.127	0.256	0.390	0.531	0.685	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
22	0.127	0.256	0.390	0.531	0.685	0.857	1.059	1.319	1.714	2.069	2.500	2.807	3.767
23	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.316	1.711	2.064	2.492	2.797	3.745
24	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.708	2.060	2.485	2.787	3.725
25	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
26	0.127	0.256	0.389	0.531	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
27	0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
28	0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.699	2.045	2.462	2.756	3.649
29	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.311	1.697	2.042	2.457	2.750	3.656
30	0.127	0.255	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.656
40	0.127	0.255	0.388	0.529	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
80	0.126	0.254	0.387	0.527	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
$\infty$	0.126	0.253	0.385	0.524	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Table 3  
Quantiles of the Chi-Square Distribution with  $\nu$  Degrees of Freedom  
( $x =$  value of  $\chi^2$  such that  $Pr(\chi^2 > x) = \alpha$ )



$\alpha$	0.990	0.975	0.950	0.900	0.100	0.050	0.025	0.010	0.001
1	0.0002	0.0010	0.0039	0.0158	2.71	3.84	5.02	6.63	10.83
2	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21	13.82
3	0.12	0.22	0.35	0.58	6.25	7.81	9.35	11.34	16.27
4	0.30	0.48	0.71	1.06	7.78	9.94	11.14	13.28	18.47
5	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09	20.52
6	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81	22.46
7	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.47	24.32
8	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	26.13
9	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	27.88
10	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	29.59
11	3.05	3.82	4.57	5.58	17.27	19.67	21.92	24.72	31.26
12	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	32.91
13	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	34.53
14	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	36.12
15	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	37.70
16	5.81	6.91	7.96	9.31	23.54	26.30	28.84	32.00	39.25
17	6.41	7.56	8.67	10.08	24.77	27.59	30.19	33.41	40.79
18	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.80	42.31
19	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	43.82
20	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	45.32
21	8.90	10.28	11.59	13.24	29.61	32.67	35.48	38.93	46.80
22	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	48.27
23	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	49.73
24	10.86	12.40	13.85	15.66	33.20	36.41	39.37	42.98	51.18
25	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	52.62
26	12.20	13.84	15.38	17.29	35.56	38.88	41.92	45.64	54.05
27	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	55.48
28	13.57	15.31	16.93	18.94	37.92	41.34	44.46	48.28	56.89
29	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	58.30
30	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	59.70

when  $\nu > 30$ , then  $\sqrt{2\chi^2 - \sqrt{2\nu - 1}}$  is approximately  $N(0,1)$ .



# 國立清華大學 104 學年度碩士班考試入學試題

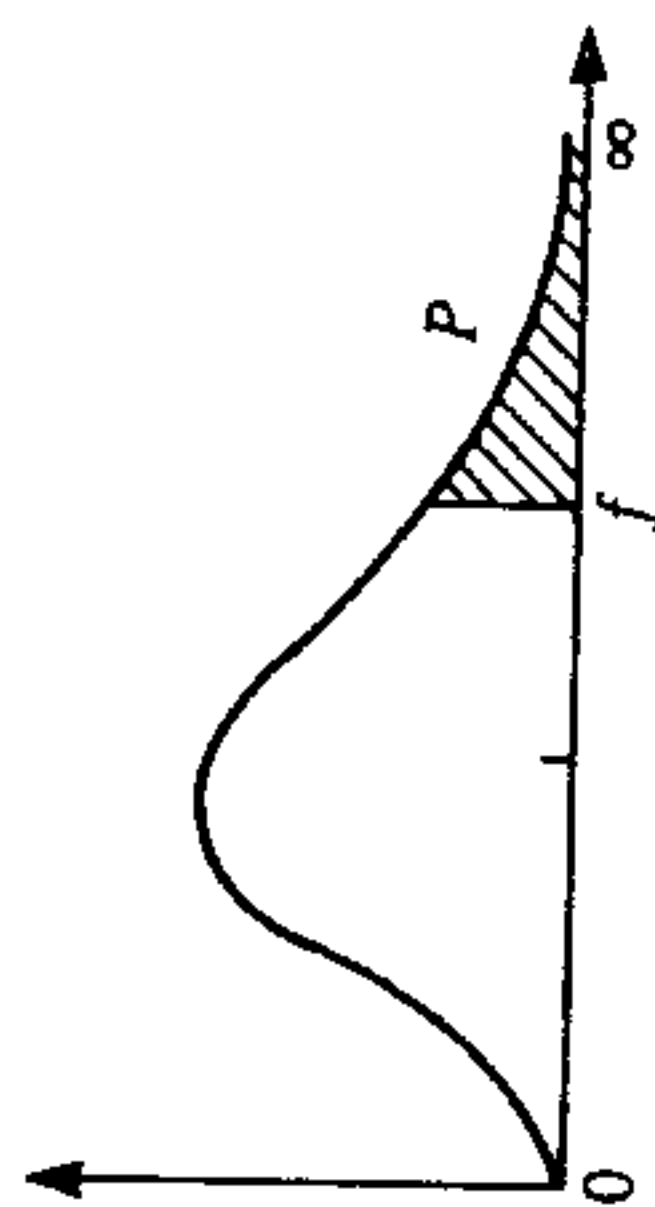
系所班組別：計量財務金融學系碩士班 甲組、乙組

考試科目（代碼）：統計學(4503)(4603)

共 9 頁，第 9 頁

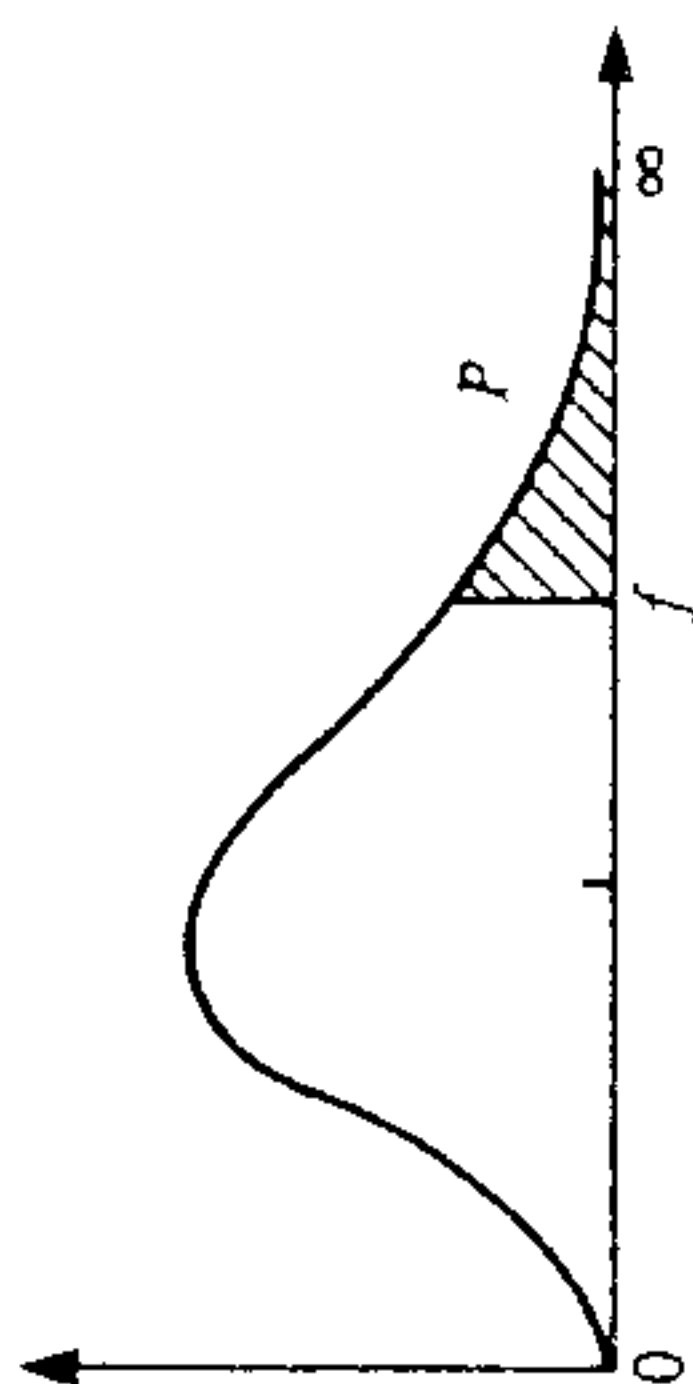
\*請在【答案卷、卡】作答

Table 5 Continued



$\nu_2$	$\nu_1 = 6$		$\nu_1 = 8$		$\nu_1 = 12$		$\nu_1 = 24$		$\nu_1 = \infty$	
	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$
1	234.0	5859	238.9	5981	243.9	6106	249.0	6234	254.3	6366
2	19.33	99.33	19.37	99.36	19.41	99.42	19.45	99.46	19.50	99.50
3	8.94	27.91	8.84	27.49	8.74	27.05	8.64	26.60	8.53	26.12
4	6.61	15.21	6.04	14.80	5.91	14.37	5.77	13.93	5.63	13.46
5	4.95	10.67	4.82	10.27	4.68	9.89	4.53	9.47	4.36	9.02
6	4.28	8.47	4.15	8.10	4.00	7.72	3.84	7.31	3.67	6.88
7	3.87	7.19	3.73	6.84	3.57	6.47	3.41	6.07	3.23	5.65
8	3.58	6.37	3.44	6.03	3.28	5.67	3.12	5.28	2.93	4.86
9	3.37	5.80	3.23	5.47	3.07	5.11	2.90	4.73	2.71	4.31
10	3.22	5.39	3.07	5.06	2.91	4.71	2.74	4.33	2.54	3.91
11	3.09	5.07	2.85	4.74	2.79	4.40	2.61	4.02	2.40	3.60
12	3.00	4.82	2.85	4.50	2.69	4.16	2.50	3.78	2.30	3.36
13	2.92	4.62	2.77	4.30	2.60	3.96	2.42	3.59	2.21	3.16
14	2.85	4.46	2.70	4.14	2.53	3.80	2.35	3.43	2.13	3.00
15	2.79	4.32	2.64	4.00	2.48	3.67	2.29	3.29	2.07	2.87
16	2.74	4.20	2.59	3.89	2.42	3.55	2.24	3.18	2.01	2.75
17	2.70	4.10	2.55	3.79	2.35	3.45	2.19	3.08	1.96	2.65
18	2.66	4.01	2.51	3.71	2.34	3.37	2.15	3.00	1.92	2.57
19	2.63	3.94	2.48	3.63	2.31	3.30	2.11	2.92	1.88	2.49
20	2.60	3.87	2.45	3.56	2.28	3.23	2.08	2.86	1.84	2.42
21	2.57	3.81	2.42	3.51	2.25	3.17	2.05	2.80	1.81	2.36
22	2.55	3.76	2.40	3.45	2.23	3.12	2.03	2.75	1.78	2.31
23	2.53	3.71	2.38	3.41	2.20	3.07	2.00	2.70	1.76	2.26
24	2.51	3.67	2.36	3.36	2.18	3.03	1.98	2.66	1.73	2.21
25	2.49	3.63	2.34	3.32	2.16	2.99	1.96	2.62	1.71	2.17
26	2.47	3.59	2.32	3.29	2.15	2.96	1.95	2.58	1.69	2.13
27	2.46	3.56	2.30	3.26	2.13	2.93	1.93	2.55	1.67	2.10
28	2.44	3.53	2.29	3.23	2.12	2.90	1.91	2.52	1.65	2.06
29	2.43	3.50	2.28	3.20	2.10	2.87	1.90	2.49	1.64	2.03
30	2.42	3.47	2.27	3.17	2.09	2.84	1.89	2.47	1.62	2.01
40	2.34	3.29	2.18	2.99	2.00	2.66	1.79	2.29	1.51	1.80
60	2.25	3.12	2.10	2.82	1.92	2.50	1.70	2.12	1.39	1.60
80	2.17	2.96	2.01	2.66	1.83	2.34	1.61	1.95	1.25	1.38
100	2.09	2.80	1.94	2.51	1.75	2.18	1.52	1.79	1.09	1.00

Table 5  
Quantiles of the Fisher-Snedecor Distribution with  $\nu_1$  and  $\nu_2$  Degrees of Freedom  
( $f =$  value of  $F$  such that  $P_r(F > f) = \alpha$ )



$\nu_2$	$\nu_1 = 1$		$\nu_1 = 2$		$\nu_1 = 3$		$\nu_1 = 4$		$\nu_1 = 5$	
	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$	$P = 0.05$	$P = 0.01$
1	161.4	4052	199.5	4999	215.7	5403	224.6	5625	230.2	5764
2	18.51	98.49	19.00	99.00	19.16	99.17	19.25	99.25	19.30	99.30
3	10.13	34.12	9.55	30.81	9.28	29.46	9.12	28.71	9.01	28.24
4	7.71	21.20	6.94	18.00	6.59	16.69	6.39	15.98	6.26	15.52
5	6.61	16.26	5.79	13.27	5.41	12.06	5.19	11.39	5.05	10.87
6	5.99	13.74	5.14	10.91	4.76	9.78	4.53	9.15	4.39	8.75
7	5.59	12.15	4.74	9.55	4.35	8.45	4.12	7.85	3.97	7.45
8	5.32	11.26	4.46	8.65	4.07	7.59	3.84	7.01	3.69	6.63
9	5.12	10.56	4.26	8.02	3.86	6.99	3.63	6.42	3.48	6.06
10	4.96	10.04	4.10	7.56	3.71	6.55	3.48	5.99	3.33	5.64
11	4.84	9.65	3.98	7.20	3.59	6.22	3.36	5.67	3.20	5.32
12	4.75	9.33	3.88	6.93	3.49	5.95	3.26	5.41	3.11	5.06
13	4.67	9.07	3.80	6.70	3.41	5.74	3.18	5.20	3.02	4.86
14	4.60	8.86	3.74	6.51	3.34	5.56	3.11	5.03	2.96	4.69
15	4.54	8.66	3.68	6.36	3.29	5.42	3.06	4.89	2.90	4.56
16	4.49	8.53	3.63	6.23	3.24	5.29	3.01	4.77	2.85	4.44
17	4.45	8.40	3.59	6.11	3.20	5.18	2.96	4.67	2.81	4.33
18	4.41	8.28	3.55	6.01	3.16	5.09	2.93	4.58	2.77	4.25
19	4.38	8.18	3.52	5.93	3.13	5.01	2.90	4.50	2.74	4.17
20	4.35	8.10	3.49	5.85	3.10	4.94	2.87	4.43	2.71	4.10
21	4.32	8.02	3.47	5.78	3.07	4.87	2.84	4.37	2.68	4.04
22	4.30	7.94	3.44	5.72	3.05	4.82	2.82	4.31	2.66	3.99
23	4.28	7.88	3.42	5.66	3.03	4.76	2.80	4.26	2.64	3.94
24	4.26	7.82	3.40	5.61	3.01	4.72	2.78	4.22	2.62	3.90
25	4.24	7.77	3.38	5.57	2.99	4.68	2.76	4.18	2.60	3.86
26	4.22	7.72	3.37	5.53	2.98	4.64	2.74	4.14	2.59	3.82
27	4.21	7.68	3.35	5.49	2.96	4.60	2.73	4.11	2.57	3.79
28	4.20	7.64	3.34	5.45	2.95	4.57	2.71	4.07	2.56	3.75
29	4.18	7.60	3.33	5.42	2.93	4.54	2.70	4.04	2.54	3.73
30	4.17	7.56	3.32	5.39	2.92	4.51	2.69	4.02	2.53	3.70
40	4.08	7.31	3.23	5.18	2.84	4.31	2.61	3.83	2.45	3.51
60	4.00	7.08	3.15	4.98	2.76	4.13	2.52	3.65	2.37	3.34
80	3.92	6.85	3.07	4.79	2.68	3.95	2.45	3.48	2.29	3.17
100	3.84	6.64	2.99	4.60	2.60	3.78	2.37	3.32	2.21	3.02