

國立清華大學 101 學年度碩士班考試入學試題

系所班組別：計量財務金融學系 乙組（財務工程組）

考試科目（代碼）：微積分（4504）

共 1 頁，第 1 頁 *請在【答案卷、卡】作答

1. (24 pts) Compute the following:

(a) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$.

(b) $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$.

(c) $\int_{-\infty}^{\infty} e^{x(2-x)} dx$.

2. (12 pts) Find all functions $f(x)$ continuous for $x > 0$, and positive real numbers a for which

$$e^x = 3 + \int_a^{x^2} f(t) dt.$$

3. (12 pts) Let $f: \mathbf{R}^2 \rightarrow \mathbf{R}$ be of class C^2 (that is, all partial derivatives of f up to order 2 exist and are continuous), and let

$$F(x, y) = f(xy, x + y).$$

Suppose that $\frac{\partial f}{\partial x}(-1, 0) = 3$, $\frac{\partial f}{\partial y}(-1, 0) = 5$, $\frac{\partial^2 f}{\partial x^2}(-1, 0) = -4$, $\frac{\partial^2 f}{\partial x \partial y}(-1, 0) = 2$, $\frac{\partial^2 f}{\partial y^2}(-1, 0) = 1$. Find $\frac{\partial F}{\partial x}(1, -1)$ and $\frac{\partial^2 F}{\partial x^2}(1, -1)$.

4. (12 pts) Suppose that $f: [a, b] \rightarrow \mathbf{R}$ is continuous with $f(a) = f(b) = 0$. If

$$x^2 f''(x) + 4x f'(x) + 2f(x) \geq 0 \quad \text{for all } x \in (a, b),$$

prove that $f(x) \leq 0$ on $[a, b]$.

5. (12 pts) If $\frac{dx}{dt} = (1-x)(1-2x)$, find $\lim_{t \rightarrow \infty} x(t)$.

6. (14 pts) Let $0 < \varepsilon < 1$.

(a) Evaluate the integral $\int_{\varepsilon}^1 (\ln x)^2 dx$.

(b) Does $\lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 (\ln x)^2 dx$ exist? If so, find its value.

7. (14 pts)

(a) Find the Maclaurin series of the function $f(x) = xe^x$.

(b) Use part (a) to find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(n-1)2^n}{n!}.$$

~特別提醒：請務必書寫計算過程，否則不予計分。~