學年度＿＿＿＿＿經濟學＿＿＿＿＿系（所）＿＿＿＿＿組碩士班入學考試

科目＿＿微積分與統計＿＿ 科目代碼＿4403＿共＿5＿頁第＿1＿頁 *請在【答案卷卡】內作答

微積分部分（共五十分）：

Instructions. There will be no point given if you do not explain how you derive your answers.

1. [9 points] Answer the following questions.
   (a) Find the critical numbers of $f(x) = x^{3/5}(4-x)$.
   (b) Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.
   (c) Find $\partial z/\partial x$ and $\partial z/\partial y$ if $xe^y + yz + ze^x = 0$.
2. [4 points] Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^4 + y^4 - 4xy + 1$.
3. [12 points] Evaluate the following integrals.
   (a) $\int \frac{5x^2+3x-2}{x^3+2x^2}dx$.
   (b) $\int x^2 \ln x dx$.
   (c) $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$.
4. [15 points] Determine whether the following series or integral converges or diverges.
   (a) $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$.
   (b) $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^7}}$.
   (c) $\int_1^{\infty} \frac{dx}{x+e^{2x}}$.
5. [10 points] In a model $C(t)$, $I(t)$, and $Y(t)$ denote respectively the consumption, investment, and national income in a country at time t. Assume that, for all t:

$$\text{(i) } C(t) + I(t) = Y(t) \quad \text{(ii) } I(t) = kC(t) \quad \text{(iii) } C(t) = aY(t) + b$$

   where a, b, and k are positive constants, with $a < 1$.
   (a) Derive the differential equation for $Y(t)$.
   (b) Solve for $Y(t)$ when $Y(0) = Y_0$, and then find the corresponding $I(t)$.
   (c) Compute $\lim_{t\to\infty}[Y(t)/I(t)]$ for $Y_0 \neq b/(1-a)$.

二、統計（共五十分）

[Instruction: Please do all **SIX** questions and show your work.]

1. (10 points) Three scores are possible in a single dart shot: 0, 1, 2. The probability distribution ($P_x(i)$) of your score (denoted as x) is given as follows:

Your score	1	2
$P_x(i)$	0.3	0.2

Your friend has a different shooting style and his score (denoted as y) distribution is as follows:

Your Friend's Score	1	2
$P_y(i)$	0.3	0.7

Each round of this competition consists of ones shot each by you and your friend. In case of a tie, anther round takes place until there is a winner for the (entire) game. We assume independence in all shots.
Note: From your perspective, there are three possible outcomes in each round: Win, Lose and Tie, with probabilities P(W), P(L), and P(T).

(a) (4 points) What is your chance of **winning the game**?

Since this is a friendly competition and the game is played at your friend's home, your friend offers the following handicap scheme for the game: You will take two shots and claim the high score. In case of a tie in the first round, subsequent rounds revert to regular single shot rounds until a winner emerges.

(b) (2 points) What is the probability that the game results in **a tie** in a single round?
(That is, P(T) =?)
(c) (2 points) What is the probability that you **win** in a single round?
(That is, P(W) =?)
(d) (2 points) What is your chance of **winning the game**?

2. (5 points) FantasiaDotCom has 10 employees. The table below shows the bonus of each employee in year 2007 and 2008. σ_1 is the standard deviation of the bonus for these employees in year 2007, and σ_2 is that in year 2008. Please show $(2\sigma_1 - \sigma_2)^2$

	Alex	*Bob*	*Cindy*	*David*	*Eric*	*Frank*	*Gina*	*Helen*	*Irene*	*Josephine*
2007	1,300	3,800	4,900	3,000	1,800	2,200	3,900	4,500	5,500	2,000
2008	2,500	7,500	9,700	5,900	3,500	4,300	7,700	8,900	10,900	3,900

3. (5 points) Suppose FORTUNE.co security firm has classified stocks into two categories using various criteria. Of the stocks they underwrite, 30% fall into the low-risk category and 70% fall in high-risk category. From the historical records, given it is low-risk stock, 20% of them have negative returns in one-year period. The corresponding figure for high-risk stocks is 50%. Assume that there has been no change in the company's business that would prevent the historical record from being a good guide to the immediate future.

If a stock gives positive return every year for two years, what is the probability the stock has been classified as a low-risk stock? (Treat different years as independent)

4. (11 points) Maria Mason wants to run a survey on the election in her town. She wants to know what percentage of voters planning to vote for Peter Ito.

(a) (5 points) How large a sample would be necessary (in the most conservative sense) to estimate the true proportion (p) that will vote for Peter Ito in a large population within ±2%, with 95% confidence?

(b) (6 points) Maria will randomly select N people and conduct the survey. Let's assume that X is the number of people (from N) supporting Mr. Ito. What is the probability distribution for X? Please show the following:

(i) The name of the probability distribution

(ii) The probability mass function

(iii) The parameter(s) for this probability mass function.

5. (5 points) Melisa assumes that wealth is linearly related to heritage and regular income. Based on the following table, she runs a regression.

Wealth(Y_i)	heritage (X_{1i})	income (X_{2i})
65000	10000	1010
70000	8000	820
90000	12000	1275
95000	14000	1425
110000	16000	1655
115000	18000	1870
120000	20000	2060
140000	22000	2205
150000	26000	2700
155000	24000	2500

The regression result is as follows:

$$\hat{Y}_i = 24647.92 + 1.58X_{1i} + 33.94X_{2i}$$
$$(6783.81) \quad (8.21) \quad (79.34)$$

$$R^2 = 0.9630 \quad \overline{R}^2 = 0.9525 \quad F(2,7) = 91.17 \quad df = 7$$

Wealth	coefficient	Std Err	t	P>|t|	95% confidence interval
Constant	24647.92	6783.814	3.63	0.008	[8606.753 40689.09]
heritage	1.581295	8.21214	0.19	0.853	[-17.83733 20.99992]
income	33.9441	79.34191	0.43	0.682	[-153.6697 221.5579]

Obviously, there is a problem of this regression. Please indicate what the problem is and show what are the evidence you based upon to conclude that there is a problem.

6. (14 points) Use the Z table at the end to answer the following questions:

(a) (6 points) Assuming that the length of fox follows the normal distribution N(μ, σ) and σ is the standard deviation. For a male fox, its length M~N(15, 4), whereas that for a female fox is F~N(10, 1.25). Assume that M and F are independent. If the length of a male coyote is 3M, and the length of a female coyote is 4F-5, what is the probability that the male coyote is 4 inches longer than the female coyote if you saw a pair of coyotes wandering in the forest when you visit Yellow Stone National Park? (Use only up to 2 decimal point for looking up the table)

(b) (8 points) You see a sign on the side of the road introducing the adult wolves in this area: The sample mean for male wolves is 20 whereas that of female wolves is 18. They sampled 50 male and 50 female wolves when conducting this survey. Let's assume both populations are approximately normal with equal standard deviation (σ=5) and the length of male and female wolves follow independent distributions. Is there a difference in length between male and female wolves? (Please designate the level of significance to be 0.05) Please show (i) the null and alternative hypothesis, (ii) the test statistics, and (iii) the conclusion for the test.

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817