

國立清華大學命題紙

96 學年度 經濟學 系(所) _____ 組碩士班入學考試

科目 微積分與統計 科目代碼 4703 共 3 頁第 1 頁 *請在【答案卷卡】內作答

Part: Calculus

There are five questions in this part. Each is worth 10 points. There are several sub-questions, whose points are indicated in the brackets at the end of each.

1. Continuous Functions. Please indicate whether the following functions are continuous, and PROVE.

A. $f(x) = \frac{\sin x}{|x|}$ (5 points) B. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{1-x^2}, & 0 \leq x \leq 1 \\ x-1, & x > 1 \end{cases}$ (5 points)

2. Differentiability.

A. Please fill in the relevant term in the bracket in the following definition: We say a function

$f: R \rightarrow R$ is differentiable at a point x if $f'(x) = \underline{\hspace{2cm}}$ exists. (5 points)

B. Use the definition above to PROVE $f(x) = x^2$ is differentiable at $x = 0$. (5 points)

3. Please solve the following two integrals.

A. $\int x \ln(x+5) dx$ (5 points) B. $\int_0^{\infty} x^{10} e^{-x} dx$ (5 points)

4. Use Lagrange multiplier method to find the point (x, y) on the curve $x^2 y = 2$ that is the closest to the origin. (10 points)

5. Implicit functions.

A. Suppose $g(x, y, z) = 0$ and $\frac{\partial g}{\partial z} \neq 0$; prove that $\frac{\partial z}{\partial y} = -\frac{\frac{\partial g}{\partial y}}{\frac{\partial g}{\partial z}}$. (5 points)

B. Prove if $g(x, y, z) = 0$, then $\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial x} = -1$. Do you need some sufficient conditions to insure the equality? (5 points)

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二、統計 (共五十分)

[Instructions: Please do all **FIVE** questions and show all your work.]

1. [10 points] There is an experiment for which there are three outcomes, A , B , and C , with respective probabilities p_A , p_B , and $p_C = 1 - p_A - p_B$. If we try to do this experiment repeatedly, what is the probability that A occurs before B does? Assume that $p_C \neq 0$.
2. [10 points] A certain river floods every year. Suppose that the low-water mark is set at 1 and the high-water mark Y has distribution function

$$F_Y(y) = \Pr(Y \leq y) = 1 - \frac{1}{y^2}, \quad 1 \leq y < \infty.$$

- (a) Find $f_Y(y)$, the pdf of Y .
 - (b) If the low-water mark is reset to 0 and we use a unit of measurement that is $1/10$ of that given previously, the high-water mark becomes $Z = 10(Y - 1)$. Find $F_Z(z)$.
3. [10 points] Let X_i , $i = 1, 2, 3$, be independent with $\mathcal{N}(i, i^2)$. For each of the following situations, use the X_i 's to construct a statistic with the indicated distribution.
 - (a) χ^2 distribution with 3 degrees of freedom
 - (b) t distribution with 2 degrees of freedom
 4. [10 points] Does a distribution exist for which

$$M_X(t) = \frac{t}{1-t},$$

where $M_X(t)$ is the moment generating function and $|t| < 1$? If yes, find it. If no, prove it.

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5. [10 points] In the simple regression model:

$$Y_i = \alpha + \beta X_i + \varepsilon_i,$$

are the following statements correct? Fully explain why or why not?

- (a) If the X -values have a small sample variance, the OLS estimators $\hat{\alpha}$ and $\hat{\beta}$ will be less precisely estimated.
- (b) If the errors ε_i are serially correlated or heteroskedastic, the OLS estimators $\hat{\alpha}$ and $\hat{\beta}$ will still be unbiased and consistent, but not efficient.