國 立 清 華 大 學 命 題 紙

95 學年度 經濟學 系（所） 一般 組碩士班入學考試

科目 微積分與統計(微積分部分) 科目代碼 5003 共 4 頁第 1 頁 *請在【答案卷卡】內作答

I. Evaluate (5 points each question)

(a) $I = \lim_{k\to\infty} I_k,\quad I_k = \int_1^{\infty}\left(\frac{k}{x} - \frac{k^2}{1+kx}\right)dx$.

(b) $J = \int_0^a dx \int_0^{a-\sqrt{(a^2-x^2)}} \frac{xe^y}{(y-a)^2}dy,\qquad a > 0.$

(c) $\frac{d}{dx}\left\{\int_{2x}^{ex} \ln(xt)dt\right\}\qquad$ at $x = 1$.

II. Let $f: X \subset R^n \to R.$

(a) Give the definition that f is a homogeneous function of degree k in x, where $x = (x_1,\ldots,x_n)$. (3 points)

(b) Show that $f_i(x) = \partial f(x)/\partial x_i$ $(i = 1,\ldots,n)$ is homogeneous of degree k-1 in x. (4 points)

(c) Prove that $\sum_1^n f_i(x)x_i = kf(x)$ **if and only if** f(x) is homogeneous of degree k in x. (6 points)

(d) Based on the homogeneous function f, give the definition of a homothetic function $g: X \subset R^n \to R.$ (3 points)

(e) Show that any homogeneous function is a homothetic function, but the converse is not true. (4 points)

III. A firm is selling a product y to two groups of consumers whose demand functions are $y_1 = p_1^{-2}$ and $y_2 = p_2^{-3}$, respectively; p_1 and p_2 are the corresponding prices, and $y = y_1 + y_2$. The firm's cost function is $c(y_1 + y_2) = 0.6(y_1 + y_2)$. (5 points each question)

(a) Find the firm's profit-maximizing levels of y_1 and y_2 if it can charge different prices in the two markets.

(b) Find the firm's profit-maximizing levels of y_1 and y_2 if it must charge the same price in both markets.

(c) Suppose the government decides to impose a specific tax on the firm's output. How would the tax affect the optimal y_1 and y_2 obtained in (a) and (b).

Please provide necessary work for your answers.

1. In a certain community, 8 percent of all adults over 50 have diabetes. If a health service in this community correctly diagnoses 95 percent of all persons with diabetes as having the disease and incorrectly diagnoses 2 percent of all persons without diabetes as having the disease, find the probabilities that

a. [5 points] the community health service will diagnose an adult over 50 as having diabetes;

b. [5 points] a person over 50 diagnosed by the health service as having diabetes actually has the disease.

2. The government took a survey on businesses to acquire their opinions about the imposition of a minimum tax. The government obtained the following responses by randomly surveying 20 large enterprises, 30 medium enterprises and 50 small enterprises.

	Large	Medium	Small
Agree	5	10	15
Disagree	10	15	25
Neutral	5	5	10

a. [5 points] Test if different sizes of enterprises have the same attitude toward the minimum tax at the significance level of 0.05. (Please check the attached statistical tables.)

b. [5 points] What key assumption(s) did you made when you did the test in (a)?

3. [5 points] First, we randomly draw a sample of 25 observations with the sample mean of 81 from a normal distribution with a standard error of 5. Second, we draw a sample of 12 observations with the sample mean of 77 from another normal distribution with a standard error of 6. Test the null hypothesis: $\mu_1 = \mu_2$ against the alternative hypothesis: $\mu_1 \neq \mu_2$ at the significance level of 0.06. (Please check the attached statistical tables.)

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4. Suppose $x_1, x_2, \ldots, x_n$ are randomly drawn from the following distribution:

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } 0 < x < \infty,\ 0 < \theta < \infty \\ 0 & \text{else} \end{cases}$$

a. [5 points] Please derive the maximum likelihood estimator of θ.

b. [10 points] Prove that the sample mean, $\bar{X}$, is an unbiased estimate of θ and its variance equals $\frac{\theta^2}{n}$.

5. Suppose you have run a least squares regression (with real data) and reported that

$$\hat{\beta} = \begin{bmatrix} 5 \\ -4 \\ 2 \end{bmatrix} \text{ and } \hat{\Sigma}_{\hat{\beta}} = \hat{\sigma}^2 (X'X)^{-1} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

a. [5 points] On the basis of this information, how would you estimate β, if you believed that $\beta_1 + \beta_2 = \beta_3$ and $\beta_2 + \beta_3 = 0$?

(Indicate the method that you would use; you need not actually carry out the computations. Define any notation that you use.)

b. [5 points] What rationale would you give for your estimator?

Cumulative Areas Under the Standard Normal Distribution

z	0	1	2	3	4	5	6	7	8	9
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148

Critical Values of the Chi-Square Distribution

		Significance Level		
		.10	**.05**	**.01**
Degrees of Freedom	1	2.71	3.84	6.63
	2	4.61	5.99	9.21
	3	6.25	7.81	11.34
	4	7.78	9.49	13.28
	5	9.24	11.07	15.09
	6	10.64	12.59	16.81
	7	12.02	14.07	18.48
	8	13.36	15.51	20.09
	9	14.68	16.92	21.67
	10	15.99	18.31	23.21
	11	17.28	19.68	24.72
	12	18.55	21.03	26.22
	13	19.81	22.36	27.69
	14	21.06	23.68	29.14
	15	22.31	25.00	30.58
	16	23.54	26.30	32.00
	17	24.77	27.59	33.41
	18	25.99	28.87	34.81
	19	27.20	30.14	36.19
	20	28.41	31.41	37.57
	21	29.62	32.67	38.93
	22	30.81	33.92	40.29
	23	32.01	35.17	41.64
	24	33.20	36.42	42.98
	25	34.38	37.65	44.31
	26	35.56	38.89	45.64
	27	36.74	40.11	46.96
	28	37.92	41.34	48.28
	29	39.09	42.56	49.59
	30	40.26	43.77	50.89

Example: The 5% critical value with $df = 8$ is 15.51.
Source: This table was generated using the Stata® function invchi.