一、微積分（共五十分）

1. Please find the maximum, and minimum of the following functions with a given domain, if they exist. If there exists no maximum or minimum, please indicate.

A. (5 points) $y = \frac{x}{1+|x|}$ on $[-1, \infty)$ B. (5 points) $y = |x^3|$ on $[-2, 3)$

2. Please **illustrate using a two dimensional graph and prove** the following statements:

Suppose $X \subseteq R^n$, and f and g are functions that map X to R; f and g are continuous at a point x^0 in X. Please prove

A. (5 points) $f \bullet g$ is continuous at x^0.

B. (5 points) $h(x)=max\{f(x),g(x)\}$ is continuous at x^0.

3. (10 points) Suppose p is a constant and f is a continuous function on R, and $f(x+p)=f(x)$ for all $x \in R$.

Prove: for all $a \in R$, $\int_a^{a+p} f(x)dx$ is a constant.

4. Solve the following two integrals:

A. (5 points) $\int \frac{(1+\sqrt{u})^{\frac{1}{2}}}{\sqrt{u}} du$. B. (5 points) $\int x^n \ln ax dx, n \neq -1.$

5. A function f on R^n is quasi-concave if for any two points x^1 and x^2 in R^n, with $f(x^1) \geqq f(x^2)$,

$$f(\lambda x^1 + (1-\lambda)x^2) \geq f(x^2), \text{ for all } \lambda \in [0,1].$$

Please determine whether the following functions are quasi-concave (x_1 and x_2 are real numbers).

A. (5 points) $f(x_1, x_2) = \ln x_1 + x_2$. B. (5 points) $(x_1^2 + x_2^2)^{-\frac{1}{2}}$.

二、統計（共五十分）

[**Instructions**: Please do all **FIVE** questions and show all your work.]

1. [**5 points**] Assume that the joint pdf of random variables X and Y is given by,

$$f(x,y) = \begin{cases} 1 & \text{for } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Please calculate $\Pr(X > Y)$.

2. A husband and wife are each 70 years old. The probability that the husband will die in the next year is 0.1, and the probability that the wife will die in the next year is 0.05. The probability that the husband will die given that the wife dies is 0.4.

   (a) [**5 points**] What is the probability that at least one of them will die in the next year?

   (b) [**5 points**] What is the probability that the wife will die given that the husband has died?

3. Suppose that the duration of a spell of unemployment (in days) X can be described by a geometric distribution,

$$\Pr(X = k) = p^k(1-p),$$

where $0 < p < 1$ is a parameter and k is a non-negative integer.

   (a) [**5 points**] What is the expected duration of unemployment?

   (b) [**5 points**] What is the probability of a spell of unemployment lasting longer than K days?

4. [**10 points**] Let X_i be *i.i.d.* with $N(\theta, \theta^2)$, $i = 1, ..., n$. Please find out the maximum likelihood estimator of θ.

5. A researcher has annual data on demand for labour, L, aggregate output in current prices, Y, average wages at current prices, W, and a general price index, P, for the manufacturing sector of a certain industrialized country for the period 1973-2002. L is measured as the average number of workers employed. Y and P are measured as index numbers equal to 100 in the year 2000. Problems associated with nonstationary time series may be ignored. He fits the following regression (standard errors in parentheses; RSS is residual sum of squares):

$$\widehat{\log L} = \underset{(0.13)}{-1.32} + \underset{(0.09)}{0.42}\log Y - \underset{(0.10)}{0.34}\log W - \underset{(0.06)}{0.11}\log P, \qquad RSS = 1.99 \tag{1}$$

He next regresses L on real output, Y/P, and real wages, W/P:

$$\widehat{\log L} = \underset{(0.13)}{-2.56} + \underset{(0.07)}{0.46}\log\left(\frac{Y}{P}\right) - \underset{(0.07)}{0.32}\log\left(\frac{W}{P}\right), \qquad RSS = 2.03 \tag{2}$$

(a) **[3 points]** Given an economic interpretation of the slope coefficients in equation (2).

(b) **[5 points]** Explain why the specification in (2) is a restricted version of that in (1), stating the restriction.

(c) **[5 points]** Perform a F test of the restriction in part (b).

(d) **[2 points]** Assuming that the restriction in part (b) is valid, explain why regression in (2) is preferable to regression in (1).