國　立　清　華　大　學　命　題　紙

九十二學年度 經濟 系（所） 組碩士班研究生招生考試

科目 微積分與統計 科號 5403 共 4 頁第 1 頁 *請在試卷【答案卷】內作答

一、微積分 (共五十分)

Instructions: Answer all questions and show all calculations.

1. a. (5 points) Suppose that x and y are independent variables and

$$r = f(x,y) = x^2 + y^2,$$
$$s = g(x,y) = x - y,$$
$$t = h(x,r) = 2x + r^2,$$
$$z = m(s,t) = (s-1)(t+1).$$

Use chain rule to find $\partial z/\partial x$ at $x = y = 1$.

b. (5 points) Assume that $y(x)$ is a differentiable function of x and that $x^3 y + y^4 = 2$. Assume that $y(1) = 1$. Find $y''(1)$.

2. a. (5 points) Let f be defined by

$$f(x) = x\sin(1/x) \quad (x \neq 0),$$
$$= 0 \quad (x = 0)$$

Find $f'(0)$ if it exists, otherwise show it.

b. (5 points) If f is defined by

$$f(x) = x^2 \sin(1/x) \quad (x \neq 0),$$
$$= 0 \quad (x = 0)$$

Find $f'(0)$ if it exists, otherwise show it.

3. a. (5 points) Evaluate the definite integral $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

b. (5 points) Evaluate $\frac{d}{dx}\left(\int_{-1}^{2x} e^{xt^2} dt\right)$ at $x = 0$.

4. For any scalar k, a real-valued function $f(x_1, \cdots, x_n)$ is homogenous of degree k if $f(tx_1, \cdots, tx_n) = t^k f(x_1, \cdots, x_n)$ for all $x_1, \cdots, x_n$ and all $t > 0$. Suppose that

$f(x_1,...,x_n)$ is continuously differentiable and homogeneous of degree k, prove

a. (5 points) the first order partial derivatives of $f(x_1,\cdots,x_n)$ are homogeneous of degree $k-1$.

b. (5 points) for all $X \equiv (x_1,\cdots,x_n)$,

$$x_1\frac{\partial f}{\partial x_1}(X)+x_2\frac{\partial f}{\partial x_2}(X)+\cdots+x_n\frac{\partial f}{\partial x_n}(X)=kf(X).$$

5. (10 points) Let $f(x,y)=x^3+y^3-3x-12y+10$, find and classify the critical points of $f(x,y)$ as yielding relative maxima, relative minima, saddle point, or none of these.

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科目 微積分與統計 科號 5403 共 4 頁第 3 頁 *請在試卷【答案卷】內作答

統計考題（每題10分）

1. X and Y are discrete random variables with joint pdf

$$f(X,Y)=\frac{X+2Y}{18} \quad when\ (X,Y)=(1,1),(1,2),(2,1),(2,2)$$

$$=0 \quad for\ all\ other\ pairs\ of\ X,Y.$$

(a) Find the marginal distribution $f(X)$ and $f(Y)$.

(b) Find the conditional mean and variance of Y when $X=1$.

2. Statisticians often deal with the mean square error (MSE) of an estimator where

$$MSE(\hat{\theta})=E\left[\left(\hat{\theta}-\theta\right)^2\right].$$

Suppose we have 2 estimators of σ^2 for the random variable X distributed $N(\mu,\sigma^2)$:

(A) $\hat{\sigma}^2=\frac{1}{N}\left[\sum_{i=1}^{N}\left(X_i-\bar{X}\right)^2\right]$, (B) $S^2=\frac{1}{N}\left[\sum_{i=1}^{N}\left(X_i-\bar{X}\right)^2\right]$

Which estimator has the smaller MSE?

3. Let X have the following probability density function

$$f(X)=\pi^{1-X}(1-\pi)^X$$

Find the maximum likelihood estimator of the parameter π.

4. Consider the following formulations of the two-variable population regression function:

Model I: $Y_i=\beta_1+\beta_2X_i+u_i$

Model II: $Y_i=\alpha_1+\alpha_2\left(X_i-\bar{X}\right)+u_i$

(a) Find the estimators of β_1 and α_1. Are they identical? Are their variances identical?

(b) Find the estimators of β_2 and α_2. Are they identical?

5. Suppose you estimate the following regression equation:

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i, \quad R^2 = 0.89$$

and you find that you cannot reject the hypotheses that $\beta_1 = 0$, $\beta_2 = 0$, and $\beta_3 = 0$ on the basis of individual t – test.

(a) What do you suspect the problem may be? If this is the only difficulty with the regression equation, what properties do your least squares estimators (including $\hat{\sigma}^2$) have?

(b) In attempting to find a linear relationship among the X variables in the regression equation, you estimate the following descriptive regression:

$$X_{1i} = 6.0 + 1.1X_{2i} - 3.0X_{3i} + v_i, \qquad R^2 = 0.95$$

On the basis of this regression, you decide to drop the variable X_3 from the model. What are the implications of this?

(c) Instead of dropping the variable X_3, what will you do to correct for the problem?