國立清華大學命題紙

八十七學年度 經濟 系(所) ________ 組碩士班研究生入學考試

科目 微積分與統計 科號 5103 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

微積分：共五十分

(1) 求解下列函數的極大值與極小值 (10 分)

$$x = y^4 - 2y^2 + 2$$

(2) 求出下列微分值 (各5 分)

(i) $y = x(\ln x)^3$; 求 $\dfrac{dy}{dx}$

(ii) 若 $w = \sqrt{x^2 + y^2 + z^2}$, $x = e^r \cos s$, $y = e^r \cos s$, $z = e^s$, 求 $\dfrac{\partial w}{\partial r}$ 及 $\dfrac{\partial w}{\partial s}$

(3) 求出下列積分值 (各5 分)

(i) $\displaystyle\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

(ii) $\displaystyle\int_1^2 \int_y^{y^3} dx\,dy$

(4) 請寫出你知道的 Taylor 展開式定理 (5 分)

(5) 設

$$y^2 + y + 3x + 1 = 0$$

求 dy/dx (5 分)

(6) 請寫出下列極大化問題的 Lagrange 函數及其一、二階必要條件 (10 分)

$$\begin{aligned} &Max \quad ax + by + cz \\ &subject\ to \quad x^2 + y^2 + z^2 = 1 \end{aligned}$$

統計考題

1. (10 points) Two students attend the same statistics class at 8:00 a.m. Student A's travel time to class from home is a random variable distributed normal with a mean of 10 minutes and a variance of 25. Student B's travel time to class from home is also a normally distributed random variable with mean of 12 minutes and a variance of 36. Since both students must contend with the early morning rush hour traffic, their travel time are assumed to be correlated with $\rho = 0.75$.
Find the probability that student B will arrive to class before student A.

2. (15 points) suppose X is a random variable distributed $N(\mu, \sigma^2)$. Two estimators of σ^2 are proposed.

(A) $s^2 = (\frac{1}{n-1})\left[\sum_{i=1}^{n}(X_i - \bar{X})^2\right]$

(B) $\hat{\sigma}^2 = (\frac{1}{n})\left[\sum_{i=1}^{n}(X_i - \bar{X})^2\right]$

where n *is the sample size and* $\bar{X} = \frac{1}{n}(\sum_{i=1}^{n} X_i)$.

(1) Are s^2 and $\hat{\sigma}^2$ unbiased estimators of σ^2?
(2) Which estimator has the smaller variance?
(3) Are the estimators consistent?

3. (10 points) Short questions. Explain your answers.

(1) Two econometrics students are overheard in the library discussing their econometrics exam. Student 1: When we estimate a least squares regression from a sample of data we get a single number for each of $\hat{\beta}_0$ and $\hat{\beta}_1$. This means $\hat{\beta}_0$ and $\hat{\beta}_1$ are constants and therefore it makes no sense to talk about their variances because the variance of a constant is always zero.
How would student 2 respond?

(2) Least squares estimation of the regression equation $Y_i = \beta_0 + \beta_1 X_i + u_i$ guarantees that the sample residuals have a zero mean and that the residuals are uncorrelated with the X variable. This implies that if the $Cov(Y_i, Y_j) = 0$ and $Var(Y|X) = \sigma^2$ then least squares estimators are BLUE. True or false?

4 (15 points) A researcher is attempting to measure the effect of commuting distance to the central business district of a city on the price of suburban houses. He plans to estimate the following regression equation:

$$\ln P_i = \beta_0 + \beta_1 \ln S_i + \beta_2 D_i + u_i \qquad (1)$$

where $\ln P_i$ = logarithm of the price of the house (in thousands of dollars)

$\ln S_i$ = logarithm of the square feet of housing space

D_i = miles to the central city

However, he is concerned that increased distance from the central city results in lower housing prices but prices fall at a decreasing rate. He specifies an alternative equation:

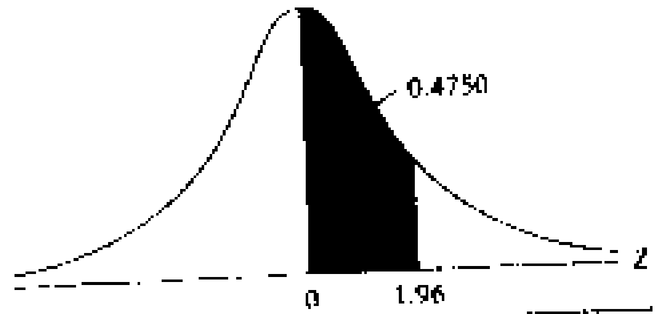
$$\ln P_i = \alpha_0 + \alpha_1 \ln S_i + \alpha_2 D_i + \alpha_3 D_i^2 + u_i^* \qquad (2)$$

1) Is multicollinearity likely to prevent him from estimating equation (2)? Explain.

2) If his concern is correct what signs should α_2 and α_3 have? How would you test the hypothesis that increased distance results in a constant proportional change in price?

3) Explain the advantages and disadvantages of using equation (1) vs. equation (2) to estimate the housing price regression.

TABLE D.1
Areas under the standardized normal distribution



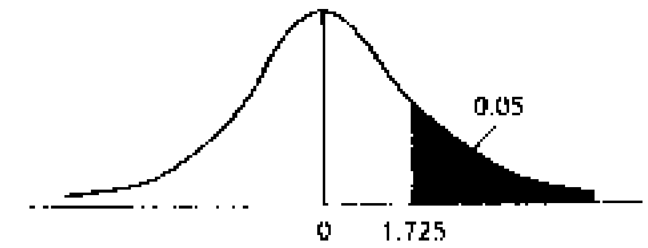
Example

$\Pr(0 \le Z \le 1.96) = 0.4750$

$\Pr(Z \ge 1.96) = 0.5 - 0.4750 = 0.025$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4571	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4814	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE D.2
Percentage points of the *t* distribution



Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for df = 20

$\Pr(|t| > 1.725) = 0.10$

df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090