八十六學年度 經濟 系（所） 組碩士班研究生入學考試

科目 微積分與統計 科號 5003 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

微積分共五題，每題 10 分。請詳細寫明計算過程。無計算過程者不計分。

1. 請解出下列兩積分值。(各 5 分)

1. $\int \frac{x}{\sqrt{1-4x^2}}dx$

2. $\int x \ln x dx$

2. 請解出下列函數的極大值。(10 分) [提示: 取單調遞增轉換。]

$$f(x_1, x_2) = e^{x_1}(x_1^2 - 2x_1x_2 + 3x_2^2)$$

3. 考慮下列函數：

$$x^2 + y^2 - 10x - 20y + 125$$

令

$$A = \{(x, y) | \ 0 \leq x \leq 100, \ 0 \leq y \leq 50\}$$

試問 A 在 f 下的映象 (image) 爲何？ (10 分)

4. 考慮下列極大化問題：

$$\begin{aligned} &\text{Max}_{(x,y)} \quad e^{x+y} \\ &\text{subject to} \quad \frac{x^2}{a} + \frac{y^2}{b} = 4 \end{aligned}$$

a. 請列出極大化的必要條件。(4 分)
b. 請列出極大化的充份條件。(4 分)
c. 請解出極大值。(2 分)

5. 請解出下列極大化問題。(10 分)

$$\begin{aligned} \text{Max}_{(x_1,x_2)} \quad & 2x_1 - x_1^2 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 3 \\ & 3x_1 - 2x_2 \leq 6 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

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統計共三題：

1. (12 points) Consider any two events A and B. Determine whether the following statements are true or false, and illustrate why.

(1) $P(A \mid B) + P(\overline{A} \mid \overline{B}) = 1$ (2) $P(A \mid B) + P(\overline{A} \mid B) = 1$

(3) $P(A \mid B) + P(A \mid \overline{B}) = 1$

2. (15 points)

(1) Suppose Y is a random variable with the following p.d.f.:

$$f(y) = \begin{cases} cy^2 & 1 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $P(Y > 3/2)$.

(c) Find Var (Y).

(2) Suppose that the joint p.d.f. of Y_1 and Y_2 is as follows:

$$f(y_1, y_2) = \begin{cases} 2y_1 e^{-y_2} & \text{for } 0 \leq y_1 \leq 1 \text{ and } 0 < y_2 < \infty \\ 0 & \text{otherwise} \end{cases}$$

Are Y_1 and Y_2 independent?

3. (15 points) Please determine whether the following statements are true or false, and explain why :

(1) If the ε_t of the regression model $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + \varepsilon_t$ follow a normal distribution with unknown variance, the least squares estimator of β_2 will follow a t-distribution with n-1 degrees of freedom.

(2) Suppose that Y_1 and Y_2 are negatively correlated. Then $\mathrm{Var}(Y_1 + Y_2) > \mathrm{Var}(Y_1 - Y_2)$

(3) For any two random variables Y_1 and Y_2, $E[\mathrm{Var}(Y_2 \mid Y_1)] + [\mathrm{Var}(E(Y_2 \mid Y_1))] = \mathrm{Var}(Y_2)$.

4. (8 points)

Suppose the p.d.f. of a random variable Y is:

$$f(y) = \begin{cases} 1/2\, y & 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

If $U = \ln Y$, find the p.d.f. of U.