國 立 清 華 大 學 命 題 紙

八十四學年度 經濟 所 組碩士班研究生入學考試
科目 微積分與統計 科號 4003 共 5 頁第 1 頁 *請在試卷【答案卷】內作答

Part I. Multiple Choice Questions (45 points). There are 9 questions. Each question is worth 5 points. **There is only one correct answer to each question.**

1. What is the Tayler series for $f(x) = e^x$ about the point $x = 1$?

A. $\sum_{i=1}^{\infty} \frac{-(x-1)^n}{n!}$

B. $\sum_{i=1}^{\infty} \frac{-e(x-1)^n}{n!}$

C. $\sum_{i=1}^{\infty} \frac{(x-1)^n}{en!}$

D. $\sum_{i=1}^{\infty} \frac{e(x-1)^n}{n!}$

E. $\sum_{i=1}^{\infty} \frac{(x-1)^n}{n!}$

2. What value of x satisfies the Mean Value Theorem for derivatives with respect to the function $f(x) = 1/x$ on the open interval $(1,2)$?

A. $\sqrt{3}$

B. $\sqrt{2}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

E. 0

3. Which of the following subsets of $(-\infty, \infty)$ is the largest domain for the real-valued function $\sqrt{x^2 - 2x}$?

A. $(-\infty, 2]$

B. $[0, \infty)$

C. $[0, 2]$

D. $(-\infty, 0] \cup [2, \infty)$

E. $(-\infty, -2] \cup [0, \infty)$

4. Let $f(x_1, x_2, x_3) = lnx_1 + x_2x_3$. The determinant of f's Hessian matrix at point $(1,1,1)$ is

A. 0

B. -1

C. 1

D. e

E. $-e$

5. Which of the following function is continuous everywhere, but has at least one point where it is not differentiable?

A. $\tan x$

B. $\frac{|x|}{x}$

C. $\sin x$

D. e^{-x}

E. $x^{1/3}$

6. $\int_0^1 \int_{2x}^2 e^{y^2} dy dx =$

A. 1

B. $\frac{1}{4}(e^4 - 1)$

C. $\frac{1}{4}(e - 1)$

D. $e^4 - 1$

E. $e - 1$

7. Let $f(x, y) = 2y^2 - 15y + x^2y - 2xy$. At which of the following critical points does f have a relative maximum?

I.$(-3, 0)$, II.$(1, 4)$, III.$(5, 0)$

A. None

B. I and II only

C. I and III only

D. II and III only

E. The correct answer is not given by A, B, C or D.

8. What is dy/dx if we know $x - y - 2\sqrt{y} = 0$?

A. 1

B. $1 + y^{-1/2}$

C. $1 + 2y^{-1/2}$

D. $2 + 2y^{-1/2}$

E. $(1 + y^{-1/2})^{-1}$

9. Let $f(x,y) = x^\alpha y^{1-\alpha}$ for some $\alpha \in (0,1)$. Which of the following statements are true?

I. f is quasi-concave. II. f is differentiable at $(0,0)$. III. f is continuous.

A. None

B. I and II only

C. I and III only

D. II and III only

E. The correct answer is not given by A, B, C or D.

Part II. There are two questions in this part.

1. Please determine whether the following statements are true, false or uncertain. Give a proof or an explanation to your claims. (No point will be given if there is no explanation to your answers).

A. If $X \subset R^n$, $Y \subset R^n$ are convex sets, then $X \cup Y$ is also convex. (5 points)

B. If A is an $n \times n$ matrix, and $b \in R^n$, then the set of x that solves $Ax = b$ is convex. (5 points)

C. The maximization problem,

$$\text{Max}_x \ f(x) \ s.t. \ x \in A, \ A \in R^n$$

must have a solution if f is differentiable and A is a closed set in R^n. (5 points)

2. The following is a maximization problem.

$$\begin{aligned} \text{Max}_{x_1,x_2} \quad & \tfrac{1}{3} ln\ x_1 + \tfrac{2}{3} ln\ x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le 3 \\ & 3x_1 - 2x_2 \le 6. \end{aligned}$$

A. Write down the Kuhn-Tucker maximization condition. (5 points)

B. Is the Kuhn-Tucker condition a sufficient condition for a maximum globally? Prove your statement. (5 points)

C. What is the pair (x_1, x_2) that solves the problem? (5 points)

1. (10 points)

(1) The political composition in T city is as follows: D party, 30 percent; K party, 50 percent; N party, 20 percent. In the last election, 82 percent of the D party members voted, 75 percent of the K party members voted, and 65 percent of the N party members also voted. Suppose we choose someone from the city at random, and discover that he did NOT vote in the last election. What is the probability that he is a member of K party?

(2) In a Lotto (樂透彩券)game, 40 balls (numbered 1 to 40) are placed in a box. 6 balls are drawn at random, without replacement. To win, you must pick the same numbers, in any order, as those drawn. If you decide to play one game this week, what is the probability of your winning?

2. (18 points) Suppose Y_1 and Y_2 have a continuous joint distribution where the joint probability density function (p.d.f.) is:

$$f(y_1, y_2) = \begin{cases} c\,(y_1 + y_2^2) & 0 \leqslant y_1 \leqslant 1, 0 \leqslant y_2 \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(1) Find the marginal p.d.f.'s of Y_1 and Y_2.

(2) Find $f(y_1 \mid y_2)$.

(2) Find $P(Y_1 \leqslant 1/2 \mid Y_2 = 1/2)$.

3. (12 points)

(1) Consider the random variable $Y \sim N(\mu, \sigma^2)$ and let $Z = e^Y$. What is the mean and variance of Z ? (Hint: Knowledge of the density of Z is not necessary).

(2) Consider the random variable X, uniformly distributed on [0, 1]. Is E[ln X] greater or less than ln E(X) ?

4. (10 points) Consider two random variables Y_1 and Y_2. Suppose that $E(Y_2 \mid Y_1) = a Y_1 + b$. If $0 < Var(Y_1) < \infty$, solve for a and b in terms of the moments and cross moments of the joint distribution of Y_1 and Y_2. i.e., find expressions for a and b in terms of $E(Y_1)$, $E(Y_2)$, $Var(Y_1)$, and $Cov(Y_1, Y_2)$.

5. (25 points) Answer the following questions **True, False or Uncertain** and very briefly explain why.

The regression model under investigation is

$$Y_t = B_0 + B_1X_{1t} + B_2X_{2t} + B_3X_{3t} + u_t \qquad (t = 1, \ldots, T)$$

where

$$E(u_t) = 0 \text{ for all } t$$

and

$$Var(u_t) = \sigma^2 \text{ for all } t$$

$$Cov(u_s, u_t) = 0 \text{ for all } s \neq t.$$

Assume the regression is being estimated by ordinary least squares (OLS) giving $\hat{B}_1, \hat{B}_2, \hat{B}_3$.

(1) If X_1 is orthogonal (i.e., the sample covariance is zero) to both X_2 and X_3, but X_3 has a positive covariance with X_2, then $\hat{B}_1$ will be unbiased but $\hat{B}_2$ and $\hat{B}_3$ will be biased in uncertain but opposite directions.

(2) If we mistakenly omit X_3 from the regression, least squares produces consistent estimators for B_1 and B_2 although they are no longer unbiased in small samples.

(3) To test the joint hypothesis that $B_2 = B_3 = 0$, we can use the F test only when u is normal.

(4) We can test whether $B_2 = B_3 = 0$ by comparing the value of R^2 with and without these restrictions being imposed.