注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 112 學年度碩士班考試入學試題

系所班組別：經濟學系

科 目 代 碼 ：4703

考 試 科 目 ：微積分與統計

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 112 學年度碩士班考試入學試題

系所班組別：經濟學系碩士班

考試科目（代碼）：微積分與統計(4703)

　　*請在【答案卷、卡】作答

PART 1. 微積分

1. Evaluate the following limits

   (a) [5 points] $\lim_{x\to 0}\frac{|x|}{x}$

   (b) [5 points] $\lim_{x\to\infty}\frac{\ln(x)}{\sqrt{x}}$

2. Calculate $\frac{dy}{dx}$ for the following functions:

   (a) [5 points] $y = ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

   (b) [5 points] $e^{xy} + x^2 - y^2 = 10$

3. Evaluate each of the following indefinite integrals:

   (a) [5 points] $\int \frac{e^{2x}}{e^x+1}dx$

   (b) [5 points] $\int \frac{1}{\sqrt{2x+1}}dx$

4. [10 points] Suppose a consumer faces two goods, X and Y, and has the utility function $U(x,y) = (x+2)^{0.5}y^{0.5}$. Suppose the prices of Good X and Y are p_x and p_y, respectively. The consumer's income is I. The consumer will maximizes his utility under the budget constraint, $p_x x + p_y y = I$. Use the Lagrange method to solve the consumer's optimal choices of Goods X and Y(in terms of p_x, p_y, I). That is,

$$\underset{\{x,y\}}{\text{Max}}\ (x+2)^{0.5}y^{0.5} \qquad \textit{subject to} \qquad p_x x + p_y y = I$$

5. [10 points] Suppose the function $f(x_1, x_2)$ is homogeneous of degree one, i.e., $f(tx_1, tx_2) = tf(x_1, x_2)$, $t \neq 0$. Prove that

$$f(x_1,x_2) = x_1 \times \frac{\partial f(x_1,x_2)}{\partial x_1} + x_2 \times \frac{\partial f(x_1,x_2)}{\partial x_2}$$

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PART 2. 統計

1. [10 points] Give the definitions of the following terms:

(a) Random variable.

(b) Sufficient statistic

2. [10 points] Let $X_1, X_2, X_3, \ldots$ be a sequence of random variable such that

$$X_n \sim \text{Binomial}(n, \lambda/n), \text{ for } n \in N, n > \lambda > 0.$$

where λ is a constant. Show that X_n converges in distribution to Poisson(λ).

3. [10 points] Consider regressing y on X which contains the intercept and one independent variable. Comparing with the original least squares estimate $\widehat{\beta^T} = (\widehat{\beta_1}, \widehat{\beta_2})$, prove that $\widehat{\beta_1}$ or $\widehat{\beta_2}$ will be the same or not after y and the independent variable are centered.

4. [10 points] Prove that uncentered coefficient of determination R^2 is invariant to a rescaling of the regressors or not (kilometers versus miles, for instance)?

5. [10 points] Can you explain the reason or the target for using the following first method rather than the second method?

(a) Ridge regression instead of standard linear regression.

(b) Lasso regularization instead of ridge regression.