

注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。


國立清華大學 111 學年度碩士班考試入學試題

系所班組別：經濟學系

科目代碼：4603

考試科目：微積分與統計

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

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系所班組別：經濟學系碩士班

考試科目（代碼）：微積分與統計（4603）

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Part 1

1. [5%] The tax T a person pays on gross income W is given by $T = a(bW + c)^p + kW$, where a, b, c , and k are positive constants, and $p > 1$. Then the average tax rate is

$$\bar{T}(W) = \frac{T}{W} = a \frac{(bW+c)^p}{W} + k$$

Find the value of W that minimizes the average tax rate.

2. [5%] Calculate $\int_{8.5}^{41} \frac{dx}{\sqrt{2x-1} - \sqrt[4]{2x-1}}$

3.

(a). [5%] Find the integral

$$I_k = \int_1^{\infty} \left(\frac{k}{x} - \frac{k^2}{1+kx} \right) dx$$

(k is a positive constant)

(b). [5%] Find the limit of I_k as $k \rightarrow \infty$, if it exists

4. [10%] Find the Taylor polynomial of order 2 about $x = 0$ for $f(x) = 5(\ln(1+x) - \sqrt{1+x})$

5. [10%] Solve the problem

max $x + 4y + z$ subject to $x^2 + y^2 + z^2 = 216$ and $x + 2y + 3z = 0$

6. [10%] For the curve given by

$$x^3 + x^2y - 2y^2 - 10y = 0,$$

find the slope and the equation for the tangent at the point $(x, y) = (2, 1)$.

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Part 2

[Instructions: Please do all questions and show your work in details.]

1. [10 pts] Suppose the joint density $f(x, y) = 1$ for $0 < x < 1, 0 < y < 1$ and $= 0$ otherwise. Please obtain $f(x|X < Y)$.
2. [10 pts] Let X be an exponential(1) random variable, and define Y to be the integer part of $X + 1$, that is

$$Y = i + 1 \text{ if and only if } i \leq X < i + 1, \quad i = 0, 1, 2, \dots$$

- (a) Find the distribution of Y . What well-known distribution does Y have?
 - (b) Find the conditional distribution of $X - 4$ given $Y \geq 5$. What is the well-known conditional distribution?
3. [30 pts] Suppose the following linear regression model:

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \varepsilon_i, \quad i = 1, \dots, n \quad (1)$$

where β_1 and β_2 are regression coefficients, and ε_i is the random error term with mean zero and $E[\varepsilon_i^2] = \sigma^2$.

- (a) Please write residual sum of square.
- (b) Please derive the variance of ordinary least squares (OLS) estimators of β_1 and β_2 .
- (c) Suppose that $X_{2i} = 1$. The OLS estimator of β_2 can be expressed as $\hat{\beta}_2 = a + b\hat{\beta}_1$. Please write a and b explicitly using available data, say $\{X_i, Y_i\}$.
- (d) Suppose that ε_i is observable. What is the unbiased estimator of σ^2 ?
- (e) As in regression model (1), ε_i is usually not observable. What is the OLS residual variance? [Hint: using fitted values]
- (f) As in the scenario of part (c), the estimator $\tilde{\beta}_1 = \sum_{i=1}^n Y_i X_{1i} / \sum_{i=1}^n X_{1i}^2$ is as efficient as the OLS estimator $\hat{\beta}_1$.