國立清華大學 106 學年度碩士班考試入學試題

系所班組別：經濟學系(0541)

考試科目（代碼）：微積分與統計(代碼 4103)

 *請在【答案卷、卡】作答

微積分部分

請<u>**依序**</u>回答問題，並於答案前 <u>**清楚標明題號**</u>。

共 50 分。每題的配分均標於題前。

1. 考慮一函數 $f: R \to R$，

(a)（5 分）請寫下 f 在 R 上是連續函數的定義。

(b)（5 分）請用你寫下的定義，證明 $f(x)=x/2$ 在 R 上是連續函數。

(c)（5 分）請寫下 f 在 R 上是可微分函數的定義。

(d)（5 分）如果 f 在 R 上可微分，f 一定是在 R 上的連續函數嗎？如果是，請證明；如果不是，請舉反例。

2. 考慮下列極大化問題：極大化 x_1x_2，受限制於 $(1-x_1-x_2)^3=0$。

(a)（5 分）在<u>**不改變限制式形狀**</u>下，請寫下 Langrange 方法的一階條件。

(b)（5 分）請用(a)的一階條件解出極大值解。如果一階條件無解，但你相信有解，但解不出來，請寫出你相信有解的理由，以及一階條件無解的原因。

3. (10 分) 請用部分積分法 Integration by parts，解出不定積分

$$\int \left(\frac{\ln x}{x}\right)^2 dx$$

4. (10 分) 以下是*微積分基本定理*：假設 *f(x)* 在 *[a,b]* 區間連續，令

$$G(x)=\int_a^x f(t)\,dt$$

則 ${dG}/{dx}(x)=f(x)$。請用「積分代表面積」的想法，解釋這定理。

國立清華大學 106 學年度碩士班考試入學試題

系所班組別：經濟學系

考試科目（代碼）：微積分與統計

共 3 頁，第 2 頁　　*請在【答案卷、卡】作答

[**Instructions**: Please do all questions and show your work in details.]

1. **[5 pts]** Chien-Ming Wang is applying to a master program in Economics. Based on how his friends have fared, he estimates that his probability of being accepted at X is 0.7 and at Y is 0.4. He also suspects there is a 75% chance that at least one of his applications will be rejected. What is the probability that he gets at least one acceptance?

2. **[5 pts]** A standard poker deck is shuffled and the card on top is removed. What is the probability that the second card is an ace?

3. **[5 pts]** Let $\bar{X}$ be the observed mean of a random sample of size n from a distribution having mean μ and known variance σ^2. Find n so that $(\bar{X}-\frac{\sigma}{4}, \bar{X}+\frac{\sigma}{4})$ is an approximate 95% confidence interval for μ.

4. **[10 pts]** Let X be a normal random variable $\mathcal{N}(\mu,\sigma^2)$. Consider a truncated normal distribution $x > a$. The distribution of such truncated distribution is $f(x|x>a)$.

   (a) Please write down $f(x|x>a)$ in terms of $\phi(.)$ and $\Phi(.)$, where $\phi(.)$ is the pdf and $\Phi(.)$ is cdf of the standard normal distribution $\mathcal{N}(0,1)$.

   (b) Suppose that Y has a uniform distribution $\mathcal{U}(0,1)$. Find the mean and variance for the truncated distribution $y > 1/3$.

5. **[25 pts]** Consider a simple linear regression model:

$$Y_i = \alpha + \beta X_i + \varepsilon_i, \tag{1}$$

   where $\varepsilon_i|X_i \sim \mathcal{N}(0,\sigma^2)$.

   (a) Please write the OLS estimator $\hat{\beta}$ as a weighted average of outcome variable Y_i. Please also specify the weight explicitly.

   (b) We can show that conditional mean of the OLS estimator $\hat{\beta}$ is β, i.e., $\mathrm{E}[\hat{\beta}|X] = \beta$. Can we also claim that unconditional $\hat{\beta}$ is unbiased, i.e., $\mathrm{E}[\hat{\beta}] = \beta$?

(c) The goodness of fit measure is called R-square, R^2, which is of the form:

$$R^2 = \hat{\beta}^2 \times \Psi,$$

where $\hat{\beta}$ is the OLS estimator. Please write Ψ explicitly.

(d) For a general definition of the F-distribution, please derive the F-statistic for the OLS slope coefficient estimator $\hat{\beta}$, i.e., testing $H_o : \beta = 0$. Note that F-statistic has a form of:

$$F = \frac{\mathcal{X}_1^2/n_1}{\mathcal{X}_2^2/n_2}.$$

Please write $\mathcal{X}_1^2/n_1$ (related to $\hat{\beta}$) and $\mathcal{X}_2^2/n_2$ (related to $\hat{\sigma}^2$).

(e) Suppose that $\alpha = 0$, the estimator $\hat{\beta}$

$$\hat{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2},$$

where x_i and y_i are in deviation forms, is the best linear unbiased estimator (BLUE). Please comment!