國立清華大學 104 學年度碩士班考試入學試題

系所班組別： 經濟學碩士班

考試科目（代碼）： 微積分與統計 (4003)

 *請在【答案卷、卡】作答

Part I. 微積分

1. a. (5%) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$ converges or diverges. If it converges, derive the value it converges to.

   b. (5%) Evaluate $\int e^{(x^2)}\,dx$ as an infinite series.

2. (10%) A firm's production function is given as follows:

$$Q = 36u^{1/2}v^{1/3}w^{1/4},$$

where u, v and w are the levels of the firm's three inputs and Q is the firm's output. Given that the firm's production budget is \$78,000, it is observed that the levels of the three inputs that maximize the firm's output are: u^*=1440, v^*=1200, w^*=1800, respectively. Solve for the corresponding prices for the three inputs.

3. a. (5%) Use differentials (or, equivalently, a linear approximation) to estimate the number: $\ln(1.07)$.

   b. (5%) Derive the Taylor polynomial of degree two for $f(x) = \sqrt{x+1}$ around $x = 0$. Then use it to estimate $\sqrt{1.1}$. What is the approximation error?

4. a. (5%) For $f(x) = 2x^3 + 4x^2 + 3x - 7$, show that there exists a number k such that $f(k) = 20$.

   b. (5%) Evaluate the following limit, if it exists: $\lim_{x\to 4} \frac{2x^3-128}{\sqrt{x}-2}$.

5. a. (5%) Let f be a function defined by $f(x) = \ln\left(\sqrt{x^2+2}\right)$. Calculate the first derivative and the second derivative. Then determine the (global or local) maximum and/or minimum values, if they exist.

   b. (5%) Evaluate the definite integral: $\int_0^4 f(x)dx$ where

$$f(x) = \begin{cases} x^2 + 2x + 3 & \text{when } 0 \le x \le 1 \\ \sqrt{4x} & \text{when } 1 < x \le 4 \end{cases}$$

Part II. 統計

[**Instructions**: Please do all questions and show your work in details.]

1. Consider the following regression to estimate the return to schooling:

$$\ln wage_i = \beta_o + \beta_1 edu_i + \beta_2 mot_i + \varepsilon_i, \quad i = 1, ..., n,$$

where $\texttt{lnwage}_i$ denotes the log-wage for individual i, $\texttt{edu}_i$ is the education level, and $\texttt{mot}_i$ represents the unobserved motivation.

(a) **[5 pts]** What is the consequence if you regress `lnwage` on a `constant` and `edu` to estimate the return to schooling (i.e., β_1)? It is reasonable to think that a well-motivated person tends to pursue a higher education level than a less-motivated individual.

(b) **[5 pts]** Now consider the following simple model:

$$\ln wage_i = \beta_o + \beta_1 D_i + u_i, \quad i = 1, ..., n,$$

where $D_i = 1$ for females and $D_i = 0$ for males. What is the ordinary least squares estimator (OLS) of β_1? Express $\hat{\beta}_1$ using the notation specified in this question.

(c) **[5 pts]** As in part (b), how do you interpret the regression coefficient β_o in this particular wage regression?

2. **[10 pts]** Cauchy-Schwartz inequality (CSI) says that:

$$(\mathrm{E}[XY])^2 \leq \mathrm{E}[X^2]\mathrm{E}[Y^2].$$

Please use CSI to show that correlation coefficient between random variables W and Z is in-between zero and one. Note that $\mathrm{E}[W] = \mu_w$, $\mathrm{var}[W] = \sigma_w^2$, $\mathrm{E}[Z] = \mu_z$ and $\mathrm{var}[Z] = \sigma_z^2$.

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3. A random variable X has the density function:

$$f_X(x) = \begin{cases} c(x+\sqrt{x}) & x \in [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

(a) **[5 pts]** Determine c.

(b) **[5 pts]** Compute $\mathrm{E}[X^{-1}]$.

(c) **[5 pts]** Determine the probability density function of $Y = X^2$.

4. Let $X_1, ..., X_n$ be a random sample drawing from:

$$f_X(x) = \theta x^{\theta-1}, 0 < x < 1, 0 < \theta < \infty.$$

(a) **[5 pts]** What is the expectation of X? Use this information and sample analog to obtain the method of moment estimator of θ.

(b) **[5 pts]** Compare the maximum likelihood estimator of θ with the method of moment estimator you have derived in part (a).