國立清華大學 103 學年度碩士班考試入學試題

系所班組別：經濟學系碩士班

考試科目（代碼）：微積分與統計(4103)

 *請在【答案卷、卡】作答

Part I. 微積分

1. a. [5 points] Compute the **first** derivative of the function: $\frac{\sqrt[4]{x^2-1}}{x^2+1}$.

   b. [5 points] Compute the **second** derivative of the function: $(\ln x)^3$.

2. [5 points] Find the functional expression of x_n in the sequence of $\left\{0, \frac{1}{5}, \frac{2}{11}, \frac{3}{19}, \frac{4}{29}, \frac{5}{41}, \ldots\right\}$ and evaluate the limit for this sequence, $\lim_{n\to\infty} x_n$.

3. [5 points] Derive the indefinite integral of the function: $\int \left(\frac{1}{3+5x} + e^{-x}\right) dx$

4. The following questions relate to the implicit function: $y^2 + 4x = 4xy^2$.

   a. [5 points] Compute $\frac{dy}{dx}$

   b. [5 points] Find the equation for the tangent line to the curve represented by the function above at the point $(\frac{1}{3}, 2)$.

5. [10 points] Utilize the Lagrangian method to find the values of the commodity bundle (x_1, x_2) which maximizes the Cobb-Douglas utility function

$$U(x_1, x_2) = 10x_1^{0.3}x_2^{0.7}$$

   with the budget constraint: $6x_1 + 7x_2 = 1200$. Also solve for the corresponding Lagrange multiplier.

6. [5 points] Approximate the value of $\sqrt[4]{9997}$ using differentials.

7. [5 points] The following function has a local minimum of 1/2 at $x = 2$:

$$f(x) = \frac{1}{a + x + bx^2}$$

   Find the values of a and b.

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Part II. 統計

 *請在[答案卷、卡]作答

Instructions: Please do all questions and show your work in details.]

1. The following regression equation is estimated as a production function:

$$\ln Q = 1.37 + \underset{(0.257)}{0.632} \ln K + \underset{(0.219)}{0.452} \ln L, \quad R^2 = 0.98,$$

where $\mathrm{cov}(b_K, b_L) = -0.044$. The sample size is 40.

   (a) [**5 pts**] What is the standard error of $b_K - b_L$?

   (b) [**5 pts**] Test $\beta_K = \beta_L$ at the 5% level of significance.

   (c) [**5 pts**] Test for constant returns to scale at the 5% level of significance.

2. [**10 pts**] Consider the regression model:

$$Y_i = \alpha_o + \alpha_1 X_i + u_i.$$

We have ordinary least squares (OLS) residual e_i, fitted value $\hat{Y}_i$, and estimators $\hat{\alpha}_o$ and $\hat{\alpha}_1$. Please determine if the followings are correct: (a) $\sum_{i=1}^n \hat{Y}_i e_i = 0$, (b) $\sum_{i=1}^n X_i e_i = 0$, (c) If the true coefficient of the constant term is zero, then the OLS fitted value is $\bar{Y}$, (d) ESS (explained sum of square) $= \hat{\alpha}_1^2 \sum_{i=1}^n (X_i - \bar{X})^2$ and (e) ESS $= \hat{\alpha}_1 \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$

3. Suppose that

$$X \sim \text{Uniform}[-1, 1]$$

and

$$Y = \begin{cases} -1 & \text{if } |X| < \frac{1}{2} \\ 1 & \text{if } |X| \geq \frac{1}{2} \end{cases} .$$

   (a) [**5 pts**] Compute $\Pr[Y = -1]$.

   (b) [**5 pts**] Use (a) to determine whether X and Y are correlated.

4. Suppose that a light bulb manufacturing plant produces bulbs with a mean life of 2000 hours and a standard deviation of 200 hours. An inventor claims to have developed an improved process that produces bulbs with a longer mean life and the same standard deviation. The plant manager randomly selects 100 bulbs produced by the process. She says that she will believe the inventor's claim if the sample mean life of the bulbs is greater than 2100 hours, otherwise she will conclude that the new process is no better than the old process. Let μ denote the mean of the new process. Consider the null and alternative hypothesis $H_o : \mu = 2000$ vs. $H_1 : \mu > 2000$. [**Hint:** $\Phi(5) = .9999$, $\Phi(-2.5) = .9938$]

   (a) [**5 pts**] What is the size of the plant manager's testing procedure?

   (b) [**5 pts**] Suppose that the new process is in fact better and has a mean bulb life of 2150 hours. What is the power of the plant manager's testing procedure?

   (c) [**5 pts**] What testing procedure should the plant manager use if she wants the size of her test to be 5%?