國立清華大學 100 學年度碩士班入學考試試題

系所班組別：經濟學研究所

考試科目（代碼）：微積分與統計 (4303)

[Please answer all questions and show your work in details.]

1.a. (5%) Find dy/dx if $y = u^3 - 5u + 1$ and $u = \sqrt{x}/(x+1)$.

1.b. (5%) Find the limit of $\lim_{x \to 0} \frac{\sqrt{1-x}-\sqrt{1+x}}{x}$.

2. Evaluate each of the following indefinite integrals.

a. (5%) $\int \frac{x}{\sqrt{x+8}}\, dx$

b. (5%) $\int \frac{2-\sqrt{x}}{2+\sqrt{x}}\, dx$

3.a. (5%) Use the Taylor approximation of order two to approximate $f(x) = x^{1/4}$ at $x = 1.1$.

3.b. (5%) Use the Taylor approximation of order two to approximate $F(x, y) = x^{1/4}y^{3/4}$ at $(x, y) = (1.1, 0.9)$.

4.a. (5%) Compute $(\partial y/\partial x_1)(6, 3)$ if the function $F(x_1, x_2, y) = x_1^2 - x_2^2 + y^3$.

4.b. (5%) A firm uses x hours of unskilled labor and y hours of skilled labor each day to produce $Q(x, y) = 60x^{2/3}y^{1/3}$ units of output per day. It currently employs 64 hours of unskilled labor and 27 hours of skilled labor. Please apply the *implicit function theorem* to estimate the corresponding change in unskilled labor that would keep the firm's output at its current level when it hires an additional hour and a half of skilled labor.

5.a. (5%) What will be the effect of a unit increase in a on the maximum value of $f(x; a) = -x^2 + 2ax + 4a^2$, where we maximize f with respect to x for each a?

5.b. (5%) Find and classify the critical points of $f(x, y) = y^3 + x^2 - 6xy + 3x + 6y$ as yielding relative maxima, relative minima, saddle points, or none of these.

國立清華大學 100 學年度碩士班入學考試試題

系所班組別：經濟學研究所

考試科目（代碼）：微積分與統計 (4303)

[**Instructions**: Please do all **FIVE** questions and show your work in details.]

1. Suppose that we are interested in estimating the coefficients in the simple linear regression model,

$$Y_i = \beta X_i + \varepsilon_i$$

and have a random sample of $\{Y_i, X_i\}_{i=1}^n$. Assume that $\mathrm{E}[Y_i|X_i] = \beta X_i$ and $\mathrm{var}[\varepsilon_i|X_i] = \sigma^2$. Consider the following three estimators for the slope coefficient,

$$b = \frac{\sum_{i=1}^n (X_i - \bar{X}) Y_i}{\sum_{i=1}^n (X_i - \bar{X})^2}, \quad \hat{\beta} = \frac{Y_n - Y_1}{X_n - X_1}, \text{ and } \quad \tilde{\beta} = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2}.$$

   (a) **[15 pts]** For the above three estimators, please determine whether or not the estimator is unbiased for β.

   (b) **[5 pts]** Which estimator is BLUE (Best Linear Unbiased Estimator)?

2. **[10 pts]** Let joint density of X and Y be

$$f(x,y) = \begin{cases} 1 & \text{if } \ 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}.$$

   Obtain $\Pr(X < Y)$.

3. **[5 pts]** If $\Pr(A) = 1/3$ and $\Pr(B^c) = 1/4$, can A and B be disjoint? Explain.

4. **[10 pts]** Suppose that nine random variables $X_1, ..., X_9$ form a random sample from a standard normal distribution, and let

$$Y = a(X_1 + X_2)^2 + b(X_3 + X_4 + X_5)^2 + c(X_6 + X_7 + X_8 + X_9)^2$$

   be a $\mathcal{X}^2$ distribution.

   (a) What is $(a + b + c)$?

   (b) What's the degree of freedom of the $\mathcal{X}^2$ distribution?

5. **[5 pts]** Let X_i be a random sample of size n with mean μ_X and variance σ_X^2. Consider $\bar{X} = \sum_{i=1}^n X_i/n$ and $s_X^2 = \sum_{i=1}^n (X_i - \bar{X})^2/(n-1)$. Construct an upper confidence limit $U(X_1, ..., X_n)$ for σ_X^2 such that

$$\Pr[\sigma_X^2 < U(X_1, ..., X_n)] = 0.99$$