科目：近代物理(3003) 校系所組：中大照明與顯示科技研究所甲組
交大電子物理學系丙組、物理研究所
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Please box your final answers!!

1.(15%) **Dimensional analysis.** Dimensional analysis is very useful to quickly gain some "feeling" of physical processes without any computation. (a) What are the dimensions of three fundamental constants c (speed of light), $\hbar$, and G (Newton's gravitational constant) in terms of L(Length), T(Time), and M (Mass)? (b) Construct a pure mass quantity and a pure time quantity by using $c, \hbar$, and G only, and (c) What are the mass quantity and time quantity in the unit of m_P (proton mass) and sec?

2.(15%) **Hydrogen spectrum.** The hydrogen 21-cm line (HI line) plays an important role in radio astronomy. This hyperfine transition line is due to the spin-spin interaction between proton and electron of hydrogen atom in the ($L = 0$, $n = 1$) state.
(a)(3%) What is the corresponding energy difference (in eV) between these two states? (b)(4%) Observation of this 21 cm line implies that some of the hydrogen atom must be excited. In some cases far away from any star, the space where hydrogen atoms reside is extremely cold. Why can we still see this 21 cm line coming from these area? (c)(4%) Provide a rough estimation of the population ratio of the hydrogen in hyperfine excited states to hydrogen in ground state. (d)(4%) Recently, there are proposals to map out the sky by radio telescope at frequencies of 200 MHz and below. These frequencies are within the working range of TV in your living room! If your TV receives a 200 MHz signal which originates from the HI line, what is the relative velocity (in c) of the radio source to earth?

3.(20%) **Quantum spin.** The physical observables in quantum mechanics are represented by matrices. For instance, the $L = 1$ angular momentum operators are described by 3×3 matrices with two components given explicitly below:

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Use the commutation relations to construct L_y matrix. (b) Write down the 3-component vector for the state $|L_x = 1\rangle$. (c) Now given a state

$$|\varphi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ i/\sqrt{2} \end{pmatrix},$$

evaluate the expectation values for L_x, L_y and L_z. (d) The standard deviation for an operator A is defined as $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$. For the state

注意：背面有試題

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$|\varphi\rangle$ given in the previous part, compute the product $\Delta L_x \cdot \Delta L_y$. Explain the physical meaning of your answer.

4.(20%) **Bohr's quantization rule.** Consider a non-relativistic particle of mass m moves in a spherically symmetric potential

$$V(r) = \left(\frac{r}{a}\right)^k V_0,$$

where V_0, a, k are some constants with appropriate physical dimensions. For simplicity, we only consider circular orbitals here. (a) Consider classical dynamics first. For a stable orbit of radius r, find out the speed v of the particle. (b) Now apply Bohr's quantization rule to obtain the quantized radius r_n. (c) Calculate the corresponding quantized energy E_n. (d) What happens to the quantized energy E_n in $k \to \infty$ limit? Please explain the physical meaning of your answer.

5.(10%) **Quantum box.** Use separation of variables in Cartesian coordinates to solve the energy spectrum of a particle of mass m in a *flat* quantum box with the confining potential expressed as $V(x,y,z) = V_x(x) + V_y(y) + V_z(z)$, where

$$V_\alpha(\alpha) = \begin{cases} 0, 0 < \alpha < L_\alpha \\ \infty, \text{otherwise} \end{cases}, \alpha = x, y, z \text{ and } L_x = L_y = 10L_z = L$$

Call the distinct eigen energies $E_1, E_2, E_3, E_4, \ldots$ in order of increasing energy. Find E_1, E_2 and E_3 (in terms of m and L), and determine their degeneracies.

6.(20%) **One dimensional simple harmonic oscillation (1D SHO).** Consider an electron in an 1D SHO. The time-independent Schrodinger equation for the harmonic oscillator is $H\psi_n = (-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2x^2)\psi_n = E_n\psi_n$. The SHO problem can be also solved by using an algebraic method, in which the ladder operators $\hat{a}_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x)$ are defined and used. Applying the defined raising (a_+) and lowering (a_-) operators to a SHO state yields $\hat{a}_+\psi_n = \sqrt{n+1}\psi_{n+1}$ and $\hat{a}_-\psi_n = \sqrt{n}\psi_{n-1}$, respectively. (a) Show that the SHO Hamiltonian can be expressed as $H = \hbar\omega(a_+a_- + 1/2)$. (b) The wave function of the ground state is given by $\psi_0 = (\frac{m\omega}{\pi\hbar})^{1/4}\exp(-\frac{m\omega}{2\hbar}x^2)$. Construct the wave function of the first excited state $\psi_1(x)$ from $\psi_0(x)$ by using the ladder operator technique. (c) Apply an external electric field along the $+x$ direction, $\vec{F} = +F\hat{i}$, onto the SHO system. Find the eigen energy, the mean values $\langle x\rangle$ and $\langle x^2\rangle$ for the first excited state of the SHO system subject to the electric field. (The electric charge of an electron is e.)