1 (15 points) The wave function of a particle trapped in an infinite square well potential of width $2a$ is found to be

$$\psi(x) = \begin{cases} C\left(\cos\dfrac{\pi x}{2a} + \sin\dfrac{3\pi x}{a} + \dfrac{1}{4}\cos\dfrac{3\pi x}{2a} \right), & \text{for } x \in [-a, a] \\ 0, & \text{elsewhere} \end{cases}$$

a) Calculate the coefficient C,

b) If a measurement of the total energy is made, what are the possible results of such a measurement, and what is the probability to measure each of them?

c) If I adiabatically widen the well to $x \in [-2a, 2a]$, what will be the new wave function after time $t \neq 0$?

2 (15 points) Give numerical values and appropriate units for:
a) Planck's constant,
b) Fine structure constant,,
c) Classical radius of electron,
d) Spin magnetic moment of proton,
e) Velocity of electron in the first Bohr orbit.
Express b) and c) in terms of the fundamental constants

$e, \hbar, m_e,$ and c.

3 (25 points) Calculate the shift in energy of the 1s state of hydrogen which one obtains if the proton is assumed to be a uniformly charged spherical shell of radius 10^{-13} cm rather than a point charge. Use the first-order perturbation theory.

4　(15 points) A one-dimensional potential barrier is of the shape
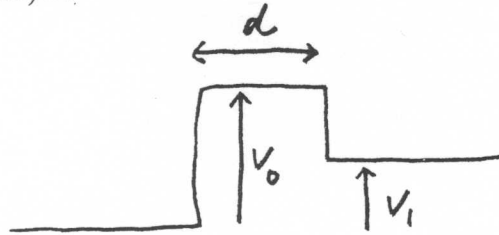
Find the transmission coefficient for particles of mass m coming from the left, with energy E $(V_1 < E < V_0)$.

5　(15 points) Electrons enter a region of uniform magnetic field B through a slit, S_1 and leave, after one semicircular turn, through slit S_2. As they enter S_1, their spins are pointed upwards $(\phi = 0)$ as shown in the diagram. Electrons have a g-factor of $g = 2 + \alpha/\pi$, and the magnetic moment is given by $\vec{\mu} = -\dfrac{e}{2mc} g \vec{s}$.
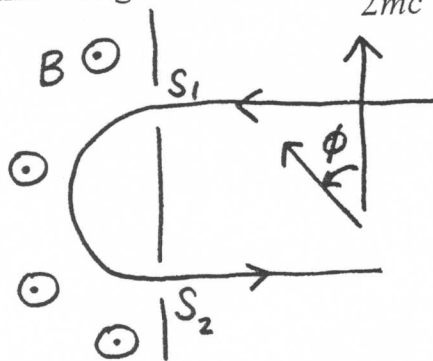
　a)　What is their spin precession frequency?
　b)　What is their cyclotron frequency?
　c)　What is the angle ϕ, which the electron spins make with the initial spin direction, when they exit through slit S_2?

6 (10 points) Determine the eigenvalues and normalized eigenvectors

of this matrix $\begin{pmatrix} 3 & 2 \\ 2 & 0 \end{pmatrix}$.

7 (5 points) What is the value of the matrix element $\left\langle \ell', m' \left| \left[\hat{L}_+, \hat{L}_- \right] \right| \ell, m \right\rangle$?