94 學年度 物理、天文 系（所） 組碩士班入學考試

科目 應用數學 科目代碼 0403 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

050乙

1 (15%)

The commutator

$$[B, A] \equiv BA - AB = \lambda I$$

of two $n \times n$ square matrices A and B is proportional to the identity matrix I.

Choose only one answer for each question.

(1) $[B, A^n] = ?$ **(A)** 0; **(B)** n; **(C)** $n\lambda A$; **(D)** $n\lambda A^{n-1}$; **(E)** $n\lambda A^n$.

(2) $e^{A+B} = ?$ **(A)** $e^A e^B$; **(B)** $e^B e^A$; **(C)** $e^A e^B e^{\frac{1}{2}[B,A]}$; **(D)** $e^A e^B e^{[B,A]}$.

(3) $e^A e^B = ?$ **(A)** e^{A+B}; **(B)** $e^B e^A$; **(C)** $e^A e^B e^{[B,A]}$; **(D)** $e^A e^B e^{-[B,A]}$.

2 (15%)

A matrix H is

$$H = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

(1) Find the eigenvalues of H.

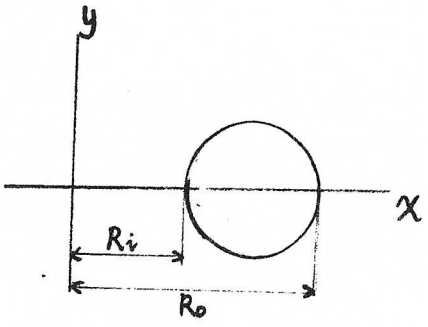
(2) Find the eigenvectors of H and the unitary matrix which diagonalizes H.

(3) Find the diagonalized matrices of H^2, H^n and e^H.

3 (20%)

(1) Calculate the surface integral

$$I_1 = \int\int \vec{r} \cdot d\vec{\sigma}$$



over the surface of a torus. As shown in the figure,

the inner and outer radii of the torus are R_i and R_o, respectively.

(2) Use Stoke's theorem to calculate the integral

$$I_2 = \int_c \vec{e}_\theta \cdot d\vec{r},$$

where c is a circle of radius R and $\vec{e}_\theta = -\sin\theta\vec{e}_x + \cos\theta\vec{e}_y$.

4 (20%)

Evaluate the integral

$$I = \int_0^\infty \frac{x^{1/2}dx}{1+x^2}$$

by contour integration and show your contour and all poles and branch cuts in the complex plane. (Hint: $\sqrt{i} = (1+i)/\sqrt{2}$.)

5 (30%)

(1) Find the general solution of the equation

$$\frac{dy}{dx} = e^{x+y+1}.$$

(2) One of the solution of the equation

$$\left\{(1-x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 2\right\} y = 0$$

is x. Find the second linearly independent solution of this equation.

(3) The function $\phi(x,y)$ is given on the plane $z=0$. Find, for $z>0$, a solution $\Psi(x,y,z)$ of Laplace's equation that reduces to $\phi(x,y)$ on the plane $z=0$. (Laplace's equation: $\nabla^2\Psi(x,y,z) = 0$.)