國立清華大學命題紙

九十二學年度 物理、天文 系（所）______組碩士班研究生招生考試

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1 (15%)

The matrices A, B, and C are given by

$$A = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 4 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

(a) Find $det(A\,B^{-1})$.

(b) Find $det(A^{-1}\,B)$.

(c) Find $det(e^{i\pi C})$.

2 (30%)

Calculate the integrals:

$$(a) \quad I_a \;=\; \int\!\!\int\!\!\int_V d\tau\,(3+r)e^r\,,$$

where $d\tau = dx\,dy\,dz$, $r = \sqrt{x^2+y^2+z^2}$ **and** V **is the volume of a sphere of radius** 1. **(Hint: Use Gauss theorem.)**

$$(b) \quad I_b \;=\; \int\!\!\int\!\!\int_{-\infty}^{+\infty} dx\,dy\,dz\,e^{-i\vec{q}\cdot\vec{r}} f(x)\delta(\vec{r}-\vec{r}_0)\,,$$

where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $\vec{r}_0 = a\vec{i} + b\vec{j} + c\vec{k}$, $\vec{q}$ **is a constant vector,** $f(x)$ **is a function of** x, **and** $\delta(\vec{r})$ **is a three-dimension Dirac** δ **function.**

$$(c) \quad I_c \;=\; \lim_{n\to\infty} \sqrt{n} \int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^n}\,.$$

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3 (20%)

Solve the differential equations

(a) $$\left(\frac{d}{dt}+2\right)\left(\frac{d}{dt}+1\right)y = 1,$$

with initial conditions $\frac{dy}{dt}|_{t=0} = y|_{t=0} = 0$.

(b) $$\frac{\partial^2 u}{\partial x^2} + \sin \pi x = \frac{\partial^2 u}{\partial t^2},$$

with initial conditions $u(x,0)=0$, $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$ **and** $u(0,t)=0$, $u(1,t)=0$.

4 (20%)

Evaluate the followings in closed forms

(a) $$2\sum_{n=1}^{\infty}\frac{x^{2n-1}}{2n-1} \quad \text{for } |x|<1.$$

(b) $$2\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right).$$

5 (15%)

Find $f(t)$ by inverting the Laplace transform

$$\frac{a^2}{p^2+a^2} = \int_0^{\infty} e^{-pt} f(t)\,dt.$$