國 立 清 華 大 學 命 題 紙

九十二學年度 物理、天文 系（所） 組碩士班研究生招生考試

科目 近代物理 科號 0401 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. A 100 Mev photon collides with a proton that is at rest. What is the maximum possible energy loss for the photon? (The rest mass of the proton is $Mc^2 = 938Mev$. You can assume it to be 1 Gev for simplicity. (20%)

2. Use the uncertainty relation to estimate the energy and the size of the ground state of a Hydrogen-like atom with nuclear charge Ze. For simplicity, you can use atomic unit , where $m_e = \hbar = e = 1$. The unit of energy and length is given by $1a.u. = 27.2eV$ and $a_{Bohr} = 0.529$å. The Hamiltonian is given by (20%)

$$H = \frac{p^2}{2m_e} - \frac{Ze^2}{r}.$$

3. At $t = 0$ a one-dimensional free particle has the wave function

$$\psi(x) = a \exp^{-\frac{x^2}{2(\delta x)^2}}$$

where a is a constant. Calculate

(a) the constant a. (5%)

(b) $< x >, < x^2 >$ (10%)

(c) $\Delta x = \sqrt{< x^2 > - < x >^2}$ (10%)

(d) $\Delta p = \sqrt{< p^2 > - < p >^2}$ by first calculate the momentum space wave function for the system. (15%)

4. The bonding and anti-bonding states of two atoms can approximately represented by the Hamiltonian matrix

$$H = \begin{bmatrix} e_1 & V \\ V^* & e_2 \end{bmatrix}.$$

where V^* is the complex conjugate of the interaction matrix element V. Calculate the energies of the bonding and the anti-bonding states. Determine the splitting of the energies in both limit cases $|V| \gg |e_1 - e_2|$ and $|V| \ll |e_1 - e_2|$. (20%)

$$\int_{-\infty}^{\infty} e^{-x^2} = \sqrt{\pi}\ ; \int_{-\infty}^{\infty} e^{-x^2 - 2ipx} = \sqrt{\pi} e^{-p^2}$$