NTHU physics Entrance Exam
Modern Physics

Useful information

$$h = 6.63 \times 10^{-34} \text{ joule} - \text{sec}$$
$$\text{electron rest mass} = 9.109 \times 10^{-31} \text{ kg}$$
$$e = 1.6 \times 10^{-19} \text{ Coulomb}$$

Problem 1 (20%) Indicate briefly the most important physics that was demonstrated in the following experiments/phenomenon

(a) Compton effect

(b) Young's Two-Slit interference experiment

(c) Davisson-Germer experiment

(d) Stern-Gerlach experiment

(e) Franck-Hertz experiment.

Problem 2 (10%) Consider the radiation in a three dimensional cavity. Let us denote $\rho(\nu)$ as the equilibrium energy density of the radiation per volume and per frequency.

(a) (5%) If we now dig a small hole with area A on the surface of the cavity, find the total power radiated out of the hole in terms of $\rho(\nu)$ (assuming that radiation incident on the hole has no reflection at all).

(b) (5%) If we put atoms with two energy levels ϵ_0 (ground state) and ϵ_1 (excited state) in the cavity to interact with the radiation and assume that the temperature is T, find the ratio of the probability for the spontaneous emission to that for the simulated emission for transition from ϵ_1 to ϵ_0.

Problem 3 (10%)

(a) (5%) Write out the configurations (for electronic orbits) for the ground states of ^{29}Cu and ^{23}V.

(b) (5%) How many lines would be expected on the detector plane of a Stern-Gerlach experiment if we use a beam of atoms with the state $^3D_2$? (assuming that the external field is strong enough to destroy the coupling between L and S) Indicate which line has the strongest intensity.

Problem 4 (10%)

(a) (5%) Write down the fundamental interactions of the nature and specify the names of the particles that mediate these interactions. If the mass of the mediate particle for an interaction is m_0, estimate the range of that interaction.

(b) (5%) Indicate the most important fundamental interactions involved in the following reactions:

$$\begin{aligned} \pi^0 &\rightarrow \gamma+\gamma, \\ {}^{27}Co^{60} &\rightarrow {}^{28}Ni^{60}+e+\overline{\nu} \end{aligned} \tag{1}$$

Problem 5 (15%) Consider the three-dimensional harmonic trap for Bose-Einstein condensate. The Hamiltonian is

$$H=\frac{1}{2m}(p_x^2+p_y^2+p_z^2)+\frac{1}{2}m\omega^2(x^2+y^2+z^2). \tag{2}$$

(a) (5%) Show that the Hamiltonian commutes with the x-component of the angular momentum

$$[H,L_x]=0. \tag{3}$$

Make use of the permutation symmetry between x, y and z to show the Hamiltonian also commutes with the other two components,

$$[H,L_y]=0, \quad [H,L_z]=0. \tag{4}$$

(b) (5%) Since L_x, L_y, L_z all commute with the Hamiltonian, can we use all l_x, l_y, l_z as good quantum numbers to label the eigenstates? Why or Why not?

(c) (5%) Near zero temperatur, bosons are trapped in the ground state. What is the rough length scale of the boson cluster?

Problem 6 (15%)

(a) (5%) The wave function of a particle at $t=0$ is $|\psi(0)\rangle=|E\rangle$, where $|E\rangle$ is an eigenstate with energy E. As time evolves, the wave function $|\psi(t)\rangle$ also varies with respect to time. What is the time derivative of the expectation of an arbitrary operator $\hat{O}$ at later time t,

$$\frac{d\langle\hat{O}\rangle}{dt}\equiv\frac{d}{dt}\langle\psi(t)|\hat{O}|\psi(t)\rangle=? \tag{5}$$

(b) (10%) Suppose the initial wave function at $t=0$ is $|\psi(0)\rangle=(1/\sqrt{2})|E_1\rangle+(1/\sqrt{2})|E_2\rangle$. The matrix elements of the operator $\hat{O}$ are $\langle E_1|\hat{O}|E_1\rangle=\langle E_2|\hat{O}|E_2\rangle=O_d$ and $\langle E_1|\hat{O}|E_2\rangle=\langle E_2|\hat{O}|E_1\rangle=O_t$, where both O_d and O_t are real numbers. Show that

$$\frac{d\langle\hat{O}\rangle}{dt}=A\sin(\omega t), \tag{6}$$

and find out A and ω explicitly.

Problem 7 (20%) Consider a one-dimensional wire with a delta-function scatter at the origin. The Hamiltonian is

$$H=\frac{p^2}{2m}+\frac{\hbar^2}{ma}\delta(x). \tag{7}$$

A right-moving wave with wave number $k>0$ can be cast into the form

$$\psi(x)=\begin{cases} e^{ikx}+re^{-ikx}, & x\le 0, \\ te^{ikx}, & x\ge 0. \end{cases} \tag{8}$$

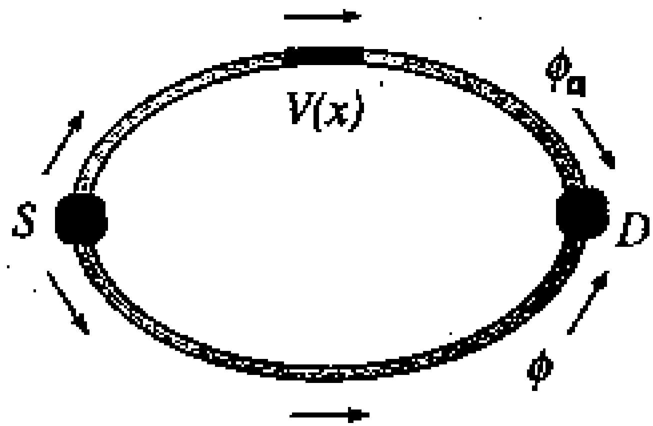

Figure 1: The interference between two quantum wires with a delta-function scatter at the upper wire.

(a) (10%) Find the reflection and transmission amplitudes r and t respectively.

(b) (5%) Calculate the current density $j(x)$ on both sides and verify the current is indeed conserved.

(c) (5%) Place a coherent source of particles at S and let particles travel through two different paths with equal length as shown in Fig. 1. Since there is a delta-function scatter at the upper path while the other path is completely free, quantum interference is expected. Calculate the phase difference $\Delta\phi = \phi_s - \phi$ at the destination D, where ϕ_s is the phase for the path with scatter and ϕ is the for the other path.