國立清華大學命題紙

八十八學年度 物理 系（所） 物理 組碩士班研究生招生考試

科目 應用數學 科號 0403 共 三 頁第 一 頁 *請在試卷【答案卷】內作答

1 (10%)

The matrix A is given by

$$A = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix},$$

where a, b, c, d, e and f are real numbers. Letting λ_i $(i = 1, 2, 3)$ be the eigenvalues of the matrix A, calculate the sums:

$$(i) \quad \sum_{i=1}^{3} \lambda_i,$$

$$(ii) \quad \sum_{i=1}^{3} \lambda_i^2,$$

in terms of $a, b, \cdots, f$.

2 (10%)

Calculate

$$T = Tr[e^{(\vec{\sigma}\cdot\vec{a})(\vec{\sigma}\cdot\vec{b})}],$$

where the components of $\vec{\sigma}$ are the three standard Pauli matrices σ_i for spin $\frac{1}{2}$, i.e.,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

3 (10%)

Evaluate in closed form the sum $f(\theta) = 1 + a\cos\theta + a^2\cos 2\theta + \cdots$.

4 (20%)

For the step function

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 0 \end{cases},$$

find

(a) its Fourier series in the interval $-\pi \leq x \leq \pi$;

(b) its Laplace transform.

5 (10%)

Find Fourier transform of the box function

$$b(x) = \begin{cases} 1, & |x| \leq \alpha \\ 0, & |x| \geq \alpha . \end{cases}$$

6 (15%)

Find a solution that is spherically symmetric and goes to zero at infinity for each of the following partial differential equations:

(a) $\nabla^2 U(\vec{r}) = -A\delta(\vec{r})$;

(b) $(\nabla^2 + k^2)U(\vec{r}) = -B\delta(\vec{r})$;

(c) $\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right) U(\vec{r}, t) = -Cf(t)\delta(\vec{r})$,

where A、B and C are constants.

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7 (10%)

Calculate the volume V of a four-dimensional unit sphere:

$$
\begin{aligned}
x_1 &= r\sin\phi_2\sin\phi_1\cos\phi, \\
x_2 &= r\sin\phi_2\sin\phi_1\sin\phi, \\
x_1 &= r\sin\phi_2\cos\phi_1, \\
x_1 &= r\cos\phi_2.
\end{aligned}
$$

8 (15%)

Evaluate the following integrals:

(a) $I_a = \int_0^{10} e^x \delta\big((x+3)(x^2-3x+2)\big)\,dx$,

where $\delta(t)$ is the Dirac's delta function;

(b) $I_c = \int_0^{\infty} e^{-x^2}\,dx$;

(c) $I_d = \int_0^{\infty} dx/(1+x^3)$.