八十七學年度 物 理 系（所）物理.應物 組碩士班研究生入學考試

應.用數學 科號 0403 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (5 points for each) Evaluate the following (a = constant):

   (a) Laplace Transform: $\mathcal{L}\left\{\int_0^t \frac{\sin au}{u} du\right\}$.

   (b) Fourier transform: $\mathcal{F}\left\{e^{-ax^2}\right\}$.

   (c) Fourier sine transform: $\mathcal{F}_s\left\{\tan^{-1}\frac{2a}{x}\right\}$.

   (d) $\int_0^\infty x^{2m}(1-x^{2n})^{-1}\,dx$, integers $n > m \geq 0$.

2. (8 points for each) Solve the following equations:

   (a) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$.

   (b) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = xe^{-2x}$.

   (c) $y^2z(\partial z/\partial x) - x^2z(\partial z/\partial y) = x^2y$.

   (d) $y(x) = x^2 + \int_0^x y(u)\sin(x-u)\,du$.

3. (8 points) Solve the heat flow in an infinitely insulated bar satisfied

$$\frac{\partial u(x,t)}{\partial t} = a^2\frac{\partial^2 u(x,t)}{\partial x^2}, \quad (a = \text{constant})$$

   with boundary conditions: $u(x,t) \to 0$, $u_x(x,t) \to 0$, at $|x| \to \infty$

   and initial condition: $u(x,0) = f(x)$.

4. (10 points) Find the principal axes for the ellipsoid (quadratic form):

$$x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1 = 5.$$

5. (10 points) Calculate the n-tuple integration:

$$I_n = \int_0^\infty dx_1 \int_0^\infty dx_2 \cdots \int_0^\infty \exp\left\{ -\sum_{i,j=1}^{n} a_{ij} x_i x_j \right\} dx_n,$$

with real constant set $\{a_{ij}\}_{i,j=1}^{n}$, $a_{ij} = a_{ji}$.

6. (10 points) Calculate the surface integration:

$$\int\int_S \nabla \times \vec{F} \cdot \hat{n}\, dA.$$

Here $\hat{n}$ is the unit normal outward of the area element dA on the upper hemisphere S: $x^2 + y^2 + z^2 = a^2$, and $\vec{F} = y\hat{i} + x(1-2z)\hat{j} - xy\hat{k}$.

7. (10 points) Riemann zeta functions are defined as:

$$\zeta(z) = \sum_{k=1}^{\infty} k^{-z}, \quad Re(z) > 1.$$

By using half-range Fourier cosine series of

$$f(x) = x^2, \quad 0 < x \leq \pi,$$

calculate $\zeta(2)$. Then integrate twice to calculate $\zeta(4)$.