八十六學年度 物理 系（所） 物理 組碩士班研究生入學考試

科目 物理數學 科號 503 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1 (16%)

(a) A Hermitian matrix H is $H = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Find the eigenvalues and the normalized eigenvectors of H and the unitary matrix which diagonalizes H.

(b) Find the explicit expression of $e^{i\theta J_z}$, where $J_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

2 (21%)

(a) Find the sum of the infinite series: $S = 1 + 2x^2 + 3x^3 + \cdots$ for $|x| < 1$.

(b) Find the Laurent expansions of function $f(z) = 1/(z^2 - 4)$ about the point $z = 2$.

(c) Find the Fourier series of the Dirac δ function $\delta(x \quad x')$ in the interval $|\quad L, L]$, where $\int_{-\infty}^{\infty} f(x)\delta(x - x')dx = f(x')$.

3 (14%)

For the Dirac δ function $\delta(x - x')$ with $x' > 0$, find

(a) its Laplace transform;

(b) its Fourier transform.

4 (21%)

(a) Find the general solutions of the equation

$$\frac{dy}{dx} = (2x - y + 3)^3 - 3.$$

(b) One of the solutions of the equation

$$\left\{(1 - x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 2\right\} y = 0$$

is $y_1(x) = x$. Find the second linearly independent solution of this equation.

(c) Find a non-trivial solution of the equation

$$\frac{d^2R(r)}{dr^2} + \frac{2}{r}\frac{dR(r)}{dr} + R(r) = 0.$$

5 (28%)

Evaluate the following integrals:

(a) $I_a = \int_{-2}^{8} \sin^2 x \delta\left(e^{-(x^2 - \pi^2/4)} - 1\right) dx$ where $\delta(t)$ is the Dirac's delta function;

(b) $I_b = \int_0^{\infty} (dx/x)(e^{-x} - e^{-2x})$;

(c) $I_c = \int_V d\tau(3 + r)e^r$, where V is the volume of a sphere of radius one;

(d) $I_d = \int_0^{\infty} dx/(4 + x^4)$.