八十五學年度 物理 系（所）物理、應物組碩士班研究生入學考試

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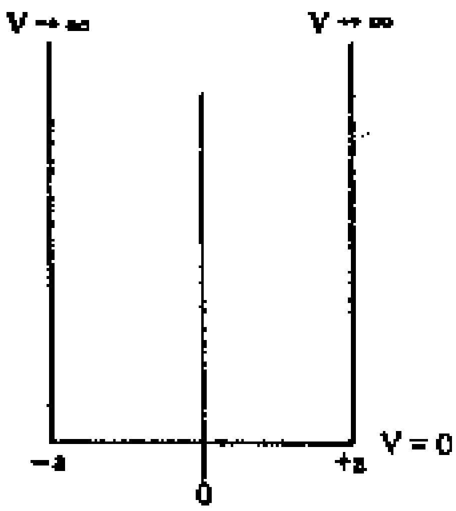
1. Consider an infinite square well. The wave function of a particle trapped in an infinite square well potential of width 2a (see figure) is found to be :

$$\psi = C\left(\cos\frac{\pi x}{2a} + \sin\frac{3\pi x}{a} + \frac{1}{4}\cos\frac{3\pi x}{2a}\right) \text{ inside the well}$$

$\psi = 0$ outside the well



   (a) Calculate the coefficient C.
   (b) If a measurement of the total energy is made, what are the possible results of such a measurement, and what is the probability to measure each of them ? (10%)

2. What are the energy levels of a particle of mass m moving in one-dimensional potential

$$V(x) = \begin{cases} +\infty & x < 0 \\ +\dfrac{m\omega^2 x^2}{2} & x > 0 \end{cases}$$

You may use the energy levels of the simple harmonic oscillator and no lengthy calculations are needed. (8%)

3. (a) State all the commutation relations among the angular momentum operators L_x, L_y, L_z and L^2.
   (b) Let $\psi_{\ell m}$ be an eigenstate of L^2 and L_z with eigenvalues $\hbar^2\ell(\ell+1)$ and $\hbar m$, respectively. Show that $\phi = (L_x - iL_y)\psi_{\ell m}$ is likewise an eigenstate of L^2 and L_z and determine the eigenvalues.
   (c) Show that if $\ell = 0$, the state $\psi_{\ell m}$ of part (b) is also an eigenstate of Lx and Ly. (10%)

4. Please write down three major observations of the photoelectric effect, that can not be explained by the classical mechanics. Also, please state the reasons that Einstein's quantum theory of the photoelectric effect explain all three observations. (10%)

5. Consider a standard 1D particle-in-the-box problem with the potential

$$V_0(x) = \begin{cases} 0 & \text{if } -a \le x \le a, \\ \infty & \text{otherwise.} \end{cases}$$

Now add a perturbing potential

$$V_1(x) = \beta\delta(x).$$

(a) Compute single particle energy levels by 1st order perturbation theory. By considering wave function symmetry, explain why some results are exact. (5%)

(b) Let ψ_n denote the n^{th} single-particle state. Now put two non-interacting particles in the box. Construct all two-particle states if particles are distinguishable. (2%)

(c) Do the same for two indistinguishable bosons. (4%)

(d) Now do the same for two identical spin-$\frac{1}{2}$ fermions. (In this case, write each wave function as a product of properly symmetrized spatial and spin wave functions.) (6%)

6. In a typical semiconductor, the wave function of zero-momentum "hole" states are made up of p-like orbitals. In the absence of spin-orbit coupling, there is a 6-fold degeneracy (3 p orbitals : p_x, p_y, p_z; 2 spins : $\uparrow, \downarrow$), with $E = E_0$. Now introduce spin-orbit interaction

$$H_{so} = \lambda \vec{L} \cdot \vec{S}.$$

(a) We need to perform angular momentum addition. What are the good quantum numbers now ? (3%)

(b) List the new eigenstates by their quantum numbers. Compute their energies. (6%)

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7. To first approximation, we can consider a positronium atom as a hydrogen atom with a very light nucleus (positron).

   (a) Find the ground state energy and radius (you can give the answers in terms of hydrogen atom values). (3%)

   (b) What physics is missing from the simple model above ? Point out which effects are relatively unimportant in hydrogen atom, but much more important in positronium atom. (5%)

8. Consider a heterostructure consisting of material A on the left side, and material B on the right. An electron in this structure obeys the effective mass equation :

$$-\frac{\hbar^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{m^*(x)}\frac{\partial}{\partial x}\psi\right] + V(x)\psi = E\psi.$$

We treat this as a 1D problem. In this case,

$$m^*(x) = \begin{cases} m_A & , x < 0 \\ m_B & , x \geq 0 \end{cases} ,$$

$$V(x) = \begin{cases} 0 & , x < 0 \\ V_B & , x \geq 0 \end{cases} .$$

   (a) What are the two boundary conditions at $x = 0$? Explain why. (Hint : B.C. different from Schrödinger equation.) (6%)

   (b) Consider the scattering of a plane wave incoming from $x = -\infty$ with $E > V_B$. Write down the proper wave functions in all regions. (5%)

   (c) What is the transmission coefficient (probability that the incoming electron is transmitted to the right side) ? What is the reflection coefficient ? (5%)

   Hint: It's useful to consider probability current densities.

$$S = \frac{\hbar}{2im^*(x)}\left(\psi^*\frac{\partial}{\partial x}\psi - \psi\frac{\partial}{\partial x}\psi^*\right)$$

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9. Explain briefly the following terms:
   (a) the uncertainty principle (4%)
   (b) the required properties of the one-dimensional eigenfunction $\psi(x)$ and its derivative $\dfrac{d\psi(x)}{dx}$ of the Shroedinger equation (4%)
   (c) the synchrotron radiation (4%)