國立清華大學命題紙

八十四學年度 物理研究 所 物理組 組碩士班研究生入學考試

科目 近代物理 科號 0401 共 三 頁第 一 頁 *請在試卷【答案卷】內作答

1. (15%) Define or explain the following terms or concepts.
   (a) van der Waals force
   (b) gyromagnetic ratio for electron
   (c) phonon
   (d) isotopes
   (e) nuclear fission
   (f) leptons

2. (15%)
   (a) Calculate the de Broglie wavelength of an electron travelling at $v = \frac{4}{5}c$.
   [Express your answer in terms of m_e, c and h]
   (b) An insulator has an optical absorption which occurs for all wavelength shorter than 1800 Å. Find the width of forbidden gap for the insulator in eV.
   (c) If the lifetime of the first excited state of the hydrogen atom is about 10^{-8} sec. Estimate the percentage spread in frequency of the photon emitted when such atom de-excites.

3. (12%) The state of a free particle moving in one dimension is specified at $t = 0$ by the wavefunction
   $$\psi(x, t = 0) = A \sin^2 kx.$$
   Find $\psi(x, t)$.
   [$E = \frac{\hbar^2 k^2}{2m}$]

4. (18%) Given $\psi(x) = Ne^{-ax^2/2}$. Calculate
   (a) N
   (b) $\langle x \rangle$, $\langle p \rangle$
   (c) $\langle x^2 \rangle$, $\langle p^2 \rangle$
   (d) Δx, Δp
   (e) The probability of finding the particle in the region $-\frac{1}{\sqrt{a}} < x < \frac{1}{\sqrt{a}}$.

5. (14%)

(a) If a particle is in eigenstate of $\hat{L}_z$ with eigenvalue $m\hbar$, show $\langle \hat{L}_x \rangle = \langle \hat{L}_y \rangle = 0$ for this state.

(b) An electron in hydrogen atom is in a state described by the wavefunction

$$\frac{1}{6}\left[4\psi_{100}(\vec{r}) + 3\psi_{211}(\vec{r}) - \psi_{210}(\vec{r}) + \sqrt{10}\,\psi_{211}(\vec{r})\right]$$

(i) What is the expectation value of energy?

(ii) What is the expectation value of $\hat{L}^2$?

(iii) What is the expectation value of $\hat{L}_z$?

[$\psi_{n\ell m}(\vec{r})$ is the wavefunction of an electron in hydrogen atom with principal quantum number n, orbital quantum number ℓ, and magnetic quantum number m. The energy eigenvalues are known to be $E_n = -\frac{1}{2}mc^2\frac{\alpha^2}{n^2} = -\frac{13.6\,eV}{n^2}$]

6. (14%)

One dimensional harmonic oscillator moves in a potential

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

It is known that its eigenvalues are given by

$$E_{n_x} = (n_x + \tfrac{1}{2})\hbar\omega, \quad n_x = 0, 1, 2, \cdots$$

with eigenfunction $\psi_{n_x}(x)$.

A two dimensional harmonic oscillator moves in a potential

$$V(x,y) = \frac{1}{2}kx^2 + \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2y^2$$

(i) Use seperation of variable method to show that the allowed energies are given by

$$E_n = (n+1)\hbar\omega, \quad n = 0, 1, 2, \cdots$$

(ii) What is the number of degenerate states for E_3? Write out the corresponding eigenfunctions.

7. (12%)

(a) At what value of Z would the $n=1$ and $n=2$ shell be full if electrons had a spin of $\frac{3}{2}$?

(b) Find the number of ways in which two particles can be distributed in five states if

(i) the particles are distinguishable,

(ii) the particles are indistinguishable and obey Bose-Einstein statistics,

(iii) the particles are indistinguishable and only particle can occupy one state.

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c = velocity of light $= 3 \cdot 10^{8}$ m/sec

h = Planck constant $= 4.14 \cdot 10^{-15}$ eV sec

$\hbar = \frac{h}{2\pi} = 6.58 \cdot 10^{-16}$ eV sec

α = fine structure constant $= \frac{e^2}{4\pi\epsilon_0 \hbar c} \cong \frac{1}{137}$

$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$, $[\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x$, $[\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$

$$\int_0^\infty x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}} \quad , \quad a > 0$$

Error function

Definition: $\operatorname{erf} x \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

x	erf x
0.5	0.520
1	0.843
1.5	0.966
2	0.995