1. (10 points) If A flips $n+1$ fair coins and B flips n fair coins. Show that the probability that A gets more heads than B is $1/2$.

2. (15 points) If the random variable X has a Beta distribution with parameters a and b, i.e. the density function of X is

$$f(x) = \begin{cases} \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}, & 0 < x < 1, \\ 0, \text{ otherwise,} \end{cases}$$

where

$$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}dx, \quad a,b > 0.$$

Find $E(X)$ and $\mathrm{Var}(X)$.

3. (15 points) Let X and Y be two independent standard normal random variables. Let (R,Θ) be the polar coordinate of (X,Y). Find the joint density funciton of (R,Θ).

4. (10 points) Choose 3 points independently at random on the perimeter of a circle. What is the probability that the triangle with the chosen points as vertices is acute?

5. (10 points) Let X and Y be two random variables with finite means. Show that $E(E(X|Y)) = E(X)$.

6. (20 points) A group of N people throw their hats into the center of room. The hats are mixed up and each person randomly selects one. Let X denote the number of people selecting their own hats. Find $E(X)$ and $\mathrm{Var}(X)$.