1. (16 points) Construct a divided-difference diagram for the function f given in the following table and write out the Newton interpolating polynomial.

x	1	3/2	0	2
f(x)	3	13/4	3	5/3

2. (16 points) Find the order of convergence of these sequences:

   (a) $x_n = n^{\frac{1}{n}}$ (b) $x_{n+1} = \tan^{-1} x_n$

3. (16 points) Given exact way of avoiding loss of significance errors in the following computations

   (a) $\log(1+x) - \log(x)$, x large.

   (b) $\tan(x) - \tan(y)$, $x \approx y$.

   (c) $(1-\cos(x))/x^2$, $x \approx 0$.

4. (16 points)

   (a) The trace of a matrix is $tr(A) = \sum_{i=1}^{n} a_{ii}$. Prove that if $\lambda_1, \cdots, \lambda_n$ are the eigenvalues of A, then the trace of A^m is $tr(A^m) = \lambda_1^m + \cdots + \lambda_n^m$.

   (b) Prove that if the eigenvalues satisfy $|\lambda_1| > |\lambda_i|$ for $i = 2, \cdots, n$, then

$$\lambda_1 = \lim_{m \to \infty} tr(A^{m+1}) \Big/ tr(A^m)$$

5. (16 points)

   (a) Find a formula of the form

$$\int_0^1 x f(x) dx \approx \sum_{i=0}^{n} w_i f(x_i)$$

   with $n = 2$ that is exact for all polynomial of degree 5.

   (b) If the integral formula

$$\int_{-1}^{1} f(x) dx \approx f(\alpha) + f(-\alpha)$$

   is to be exact for all quadratic polynomials, what value of α should be used?