國立清華大學命題紙

八十七學年度 數學系 系(所) 純粹數學 組碩士班研究生入學考試

科目 拓樸學 科號 0104 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. (10 points)

Prove or disprove that $S^1 = \{(a,b)/a^2+b^2=1\} \subsetneq R^2$ (in the usual topology) is homeomorphic with $S^2 = \{(a,b,c)/a^2+b^2+c^2=1\} \subsetneq R^3$ (in the usual topology).

2. (15 points)

Let $A \subseteq X$, let $f : A \longrightarrow Y$ be a continuous map, and let Y be a Hausdorff space. Show that if the map f may be extended to a continuous map $g : \overline{A} \longrightarrow Y$, then the map g is uniquely determined by the map f, where $\overline{A}$ is the closure of A.

3. (10 points)

Prove the union of a collection of connected sets that have a point in common is connected.

4. (15 points)

Define the quotient space $[R^2 - (0,0)]/\sim$, where $\sim$ is the equivalene relation given by $(a,b) \sim (c,d)$ if and only if $(ta,tb) = (c,d)$ for some $t \in R$. Then prove that $S^1 = \{(a,b)/a^2+b^2 = 1\} \subsetneq R^2$ (in the usual topology) is homeomorphic with this quotient space $[R^2 - (0,0)]/\sim$.

5. (10 points)

Prove that every compact subset of a topological Hausdorff space is closed.