

八十六學年度 數學系 系(所) 應 數 組碩士班研究生入學考試

科目 數 值 分 析 科號 0204 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (20 points)

For a given secondly continuously differentiable function $f(x)$, define the second order divided difference $f[x_0, x_1, x_2]$ by

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

with $f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$ and $f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$, assuming x_0, x_1, x_2 are distinct.

(a) Prove the order of the arguments x_0, x_1, x_2 does not affect the value of the divided difference $f[x_0, x_1, x_2]$.

(b) Prove that

$$f[x_0, x_1, x_2] = \frac{1}{2}f''(\xi),$$

for some ξ between the minimum and maximum of x_0, x_1 and x_2 .

Hint: use Taylor's Theorem and intermediate value Theorem.

2. (15 points)

Given exact way of avoiding loss of significance errors in the following computations

(a) $\log(1+x) - \log(x)$, for large x

(b) $\tan(x) - \tan(y)$, for $x \approx y$

(c) $[1 - \cos(x)]/x^2$, for $x \approx 0$

3. (15 points)

Show that

$$x_{n+1} = \frac{x_n(x_n^2 + 3a)}{3x_n^2 + a}, \quad n \geq 0$$

is a third order method for computing \sqrt{a} . Calculate

$$\lim_{n \rightarrow \infty} \frac{\sqrt{a} - x_{n+1}}{(\sqrt{a} - x_n)^3}$$

assuming x_0 has been chosen sufficiently close to \sqrt{a} .

4. (15 points)

Consider the function e^x on $[0, b]$ and its approximation by an interpolating polynomial. For $n \geq 1$, let $h = b/n$, $x_j = jh$, $j = 0, 1, \dots, n$; and let $p_n(x)$ be the $n+1$ degree polynomial interpolating to e^x on the nodes x_0, \dots, x_n . Prove that

$$\max_{0 \leq x \leq b} |e^x - p_n(x)| \rightarrow 0 \text{ as } n \rightarrow \infty$$

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5. (15 points)

Calculate the condition numbers $\text{cond}_p(A)$, for $p = 1, 2$ and ∞ for $A =$

$$\begin{bmatrix} 100 & 99 \\ 99 & 98 \end{bmatrix}.$$