1. (20%)

   Prove that the following statements are equivalent to each other.

   (a) X is Hausdorff topological space.

   (b) Let $p \in X$. For each $q \neq p$, there is a neighborhood $U(p)$ of p such that $q \notin \overline{U(p)}$, where $\overline{U(p)}$ is the closure of $U(p)$.

   (c) For each $p \in X$, $\cap \{\overline{U} | U$ is a neighborhood of $p\} = p$, where $\overline{U}$ is the closure of U.

   (d) The diagonal $\Delta = \{(p, p) | p \in X\}$ is closed in $X \times X$ with product topology.

2. (10%)

   Let X be a locally compact Hausdorff space that is not compact. Prove that X is dense in the one-point compactification $Y = X \cup \{\infty\}$ of X.

3. (10%)

   Let X be a T_4-space. Let E be a closed subset of X, and $f : E \to \mathbb{R}$ be continuous. Show that f can be extended to a continuous function from X to $\mathbb{R}$. (Hint: Let $g : \mathbb{R} \to (-1, 1)$ be a homeomorphism, extend $g \circ f$ to a map from X to $[-1, 1]$, and decide what to do on the set where the extension assumes the value ± 1.)

4. (10%)

   Define an equivalent relation in $X = [0, 1] \times [0, 1]$ by declaring $(s_0, t_0) \sim (s_1, t_1)$ if and only if $t_0 = t_1 > 0$.
   (a) Describe the quotient space $X/\sim$.
   (b) Prove or disprove that $X/\sim$ is Hausdorff.

5. (10%)

   Prove or disprove that a continuous map from a connected topological space to a discrete topological space must be a constant map.