1. A chest has three draws. The first contains two gold coins, the second contains a gold and a silver coin, and the third has two silver coins. A draw is chosen at random then from it a coin is chosen at random. What is the probability that the coin still remaining in the chosen draw is gold, given that the coin chosen is silver? (10 points)

2. Let $\{X_n\}$ be a sequence of independent random variables such that

$$P(X_n = 1) = P(X_n = -1) = \frac{1}{2}, \quad \forall\, n \geq 1.$$

Let $S_n = \sum_{j=1}^{n} X_j$. Show that

$$\sum_{n=1}^{\infty} P(S_n = 0) = \infty. \qquad (10 \text{ points})$$

3. Let $X$ be a Poisson random variable; i.e.,

$$P(X = n) = \frac{e^{-\lambda}\lambda^n}{n}, \quad n = 0, 1, 2, 3, \cdots, \ 0 < \lambda < \infty.$$

Find (a) $\phi(t) = E(t^X), \ t \in R^1,$

(b) $m(t) = E(e^{tX}), \ t \in R^1,$

(c) $f(t) = E(e^{itX}), \ i = \sqrt{-1}, \ t \in R^1.$

Are these functions analytic is $t$? (15 points)

4. Let $X$ and $Y$ be two independent standard normal random variables. Find the density function of $X/Y$. (10 points)

5. Let $X_1, X_2, X_3$ be three independent standard normal random variables and let $Y_1 = X_1 + X_2, \ Y_2 = X_2 + X_3$. Find the joint density function of $Y_1$ and $Y_2$. (15 points)

6. Let $\Theta_1, \Theta_2, \cdots, \Theta_n$ be $n$ independent random variables, each uniformly distributed over $[0, 2\pi)$. Let $S_n = \sum_{j=1}^{n} \exp(i\Theta_j)$, $i = \sqrt{-1}$. Find $E(|S_n|^2)$. (10 points)