1. Prove or disprove the convergence of the following series.

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^{1.2}} \cdot \qquad \text{(15 points)}$$

2. If $\sum_{n=1}^{\infty} a_n$ diverges with $a_n > 0$ for all $n$, show that $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ diverges. (20 points)

3. Let $f(x)$ be an increasing function defined on $[a, b], b > a$. Show that the discontinuities of $f(x)$ on $[a, b]$ is at most countable. (15 points)

4. Prove the mean value theorem for the derivative. Namely, show that "if $f(x)$ is a real differentiable function on $[a, b]$ and $f'(a) < \lambda < f'(b)$, then there is a point $p \in (a, b)$ such that $f'(p) = \lambda$". (15 points)

5. Suppose that $f(x)$ is a bounded real function defined on $[0, 1]$ and that $f^2(x)$ is Riemann integrable on $[0, 1]$. Does it follow that $f(x)$ is Riemann integrable on $[0, 1]$. Show your reason. (15 points)

6. Let $f(x)$ be a $C^{\infty}$-function with compact support defined on $\mathbb{R}$ such that $\int_{\mathbb{R}} f(x)dx \neq 0$. Show that there does not exist a $C^{\infty}$- function $g(x)$ with compact support such that $\frac{dg}{dx} = f$. (15 points)

7. Show that the given family of continuous functions

$$f_n(x) = \frac{x^2}{x^2 + (1 - nx)^2}, \quad \text{for} \quad x \in [0, 1] \text{ and } n \in N,$$

is not equicontinuous on $[0, 1]$. (20 points)

8. Give a metric space $X$ on which the concept of "boundedness and closedness" is not equivalent to the "compactness". (15 points)