


注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：數學系

考試科目(代碼)：高等微積分(0101)

— 作答注意事項 —

1. 請核對答案卷(卡)上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清(含未依範例畫記)致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

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共__1__頁，第__1__頁 *請在【答案卷、卡】作答

(1) (15%) If $a_1 = \sqrt{2}$, and

$$a_{n+1} = \sqrt{2 + \sqrt{a_n}}, \quad n = 1, 2, 3, \dots$$

prove that the sequence $\{a_n\}$ converges, and that $a_n < 2$ for $n = 1, 2, 3, \dots$.

(2) Define a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ by

$$f(x, y) = \begin{cases} 0, & (x, y) = (0, 0) \\ x^2 + y^2 - 2x^2y - \frac{4x^6y^2}{(x^4+y^2)^2}, & (x, y) \neq (0, 0) \end{cases}$$

(a) (5%) Prove that f is a continuous function on \mathbb{R}^2 .

(b) (15%) Prove that the restriction of f to each line through $(0, 0)$ has a local minimum at $(0, 0)$.

(c) (10%) Is $(0, 0)$ a local minimum for f ? (Please give your reasons.)

(3) (10%) Prove that the function $f(x) = \ln x$ is not uniformly continuous on $(0, 1)$.

(4) (10%) For $n = 1, 2, 3, \dots$, x real, put

$$f_n(x) = \frac{x}{1 + nx^2}.$$

Show that $\{f_n\}$ converges uniformly to a function f . Does $\{f'_n\}$ converges to f' ?

(5) (10%) Let D be the solid bounded by the cylinder $x^2 + y^2 = 4$, the plane $x + z = 6$, and the xy -plane. Find

$$\int \int_S F \cdot \vec{n} \, dS$$

where S is the boundary of D with the unit normal vector \vec{n} directed outward from D and $F(x, y, z) = (x^2 + \sin z)\mathbf{i} + (xy + \cos z)\mathbf{j} + e^y\mathbf{k}$.

(6) (a) (15%) Let $f : [0, 1] \rightarrow \mathbb{R}$ be a one-to-one continuous function and A the range of f . Show that the inverse function $f^{-1} : A \rightarrow [0, 1]$ is continuous.

(b) (10%) Please give a function $g : [0, 1] \rightarrow \mathbb{R}^2$ which is one-to-one continuous. But its inverse, which is defined on the range of g , is not continuous.