# 國立清華大學 105 學年度碩士班考試入學試題

系所班組別：數學系　應用數學組

考試科目（代碼）：高等微積分 (0201)

　*請在【答案卷】作答

1. Suppose $A$ is a connected set in $\mathbb{R}^2$ that contains $(-1, 2)$ and $(6, 5)$. Show that $A$ contains at least one point on the line $x = y$.

2. Let $f : [a, b] \to \mathbb{R}$ is an integrable function. If for every subinterval $I \subset [a, b]$ there is a point $c \in I$ such that $f(c) = 4$. Compute $\int_a^b f(x)\,dx$.

3. Find the extreme values of $f(x, y) = 2x^2 + y^2 + 2x$ on the set $\{(x, y) | x^2 + y^2 \leq 1\}$.

4. Let $f_n(x) = g(x)\,x^{2n}$, where $g$ is continous on $[0, 1]$ and $g(1) = 0$. Show that $f_n \to 0$ uniformly on $[0, 1]$.

5. Suppose $f$ is a function defined on an open set $S \subset \mathbb{R}^3$. Show that if the partial derivatives $\frac{\partial f}{\partial x_j}$ exist and are bounded on $S$, then $f$ is continous on $S$.

6. Let $f(x, y) = \frac{3x\,y^2}{x^2 + y^2}$ if $f(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Prove or disprove $f$ is differentiable at $(0, 0)$.

7. Let $f_n$ be a sequence of functions that converges uniformly to $f$ on $I \subset \mathbb{R}$ and that satisfies $|f_n(x)| \leq M$ for all $n \in \mathbb{N}$ and all $x \in I$. If $g$ is continuous on $\mathbb{R}$, show that the sequence $g \circ f_n$ converges uniformly to $g \circ f$ on $I$.

8. Let $f_n$, $g_n$, $h_n$ be sequences of functions on $\mathbb{R}$ that satisfy $f_n \leq g_n \leq h_n$ on $\mathbb{R}$ for all $n \in \mathbb{N}$. If $\sum f_n$ and $\sum h_n$ converge, show that $\sum g_n$ converges.

9. Let $f : [a, b] \to \mathbb{R}$ is a continuous function. If for each $x \in [a, b]$ there exists $y \in [a, b]$ such that $|f(y)| \leq \frac{1}{3}|f(x)|$, show that there exists a point $c \in [a, b]$ such that $f(c) = 0$.

(第9題12分，其餘每題11分）