國立清華大學 103 學年度碩士班考試入學試題

系所班組別：數學系　數學組

考試科目（代碼）：高等微積分(0101)

　　*請在【答案卷、卡】作答

1. Let $\{a_n\}$ and $\{b_n\}$ be given sequences of real numbers and let the sequence $\{c_n\}$ be defined by

$$c_1 = a_1,\ c_2 = b_1,\ \cdots,\ c_{2n-1} = a_n,\ c_{2n} = b_n,\ \cdots.$$

Show that $\{c_n\}$ is convergent, if $\{a_n\}$ and $\{b_n\}$ are convergent to L.

2. Show that if f is continous on $[0, \infty)$ and uniformly continuous on $[a, \infty)$ for some positive constant a, then f is uniformly continuous on $[0, \infty)$.

3. If $f : [a, b] \to \mathbb{R}$ is a bounded function such that $f(x) = 0$ except for x in $\{c_1, c_2, \cdots, c_n\}$ in $[a, b]$, show **by definition** that f is integrable on $[a, b]$ and that $\int_a^b f(x)\, dx = 0$.

4. Let f be continous on $[a, b]$ and assume the second derivative f'' exists on (a, b). Suppose that the graph of f and the line segment joining the points $(a, f(a))$ and $(b, f(b))$ intersect at a point $(x_0, f(x_0))$ where $a < x_0 < b$. Show that there exists a point $c \in (a, b)$ such that $f''(c) = 0$.

5. Let $f(x) = \sum_1^\infty (x+n)^{-2}$. Show that f is a continous function on $[0, \infty)$ and that $\int_0^1 f(x)\, dx = 1$. (16 points)

6. Let C be the unit circle $x^2 + y^2 = 1$, oriented counterclockwise. Compute the line integral

$$\int_C \left[\sqrt{1+x^2} - ye^{xy} + 3y\right] dx + [x^2 - xe^{xy} + \ln\ (1+y^4)]\, dy.$$

7. Prove or disprove that the equations $x^2 + y^2 + z^2 = 3,\ xy + tz = 2,\ xz + ty + e^t = 0$ can be solved for x, y and z as C^1 functions of t near $(x, y, z, t) = (-1, -2, 1, 0)$.

(Except for problem 5, each problem for 14 points.)