國立清華大學 102 學年度碩士班考試入學試題

系所班組別：數學系　應用數學組

考試科目（代碼）：線性代數 (0202)

　　*請在【答案卷、卡】作答

In the problems below, R is the field of the real numbers, under the usual addition and scalar multiplication, $M_{m\times n}(R)$ is the vector space consisting of all $m\times n$ matrices over R, and if $A\in M_{n\times n}(R)$, then $\det(A)$ is the determinant of A.

In each of problems 1-5, just give the answer as true or false without any proof or reason.

(5%) 1. $A\in M_{n\times n}(R)$, and $A^3=A$, then A is diagonalizable.

(5%) 2. If the characteristic polynomial of a square matrix splits, then this square matrix is similar to its Jordan canonical form.

(5%) 3. If $A\in M_{n\times n}(R)$, $n\geq 5$, then $\det(A^t)=-\det(A)$, where A^t is the transpose of A.

(5%) 4. The vector space $M_{3\times 4}(R)$ is isomorphic to the usual vector space R^7.

(5%) 5. If $D\in M_{n\times n}(R)$, and D can be written in the form

$$D=\begin{pmatrix} A & B \\ O & C \end{pmatrix},$$

where A and C are square matrices, and O is the zero matrix, then $\det(D)=\det(A)\det(C)$.

In each of problems 6-8, just give the answer without any proof or reason.

(10%) 6. The number of the Jordan blocks of the matrix

$$\begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{pmatrix}$$

is (　　).

(10%) 7. The rank of the matrix

$$\begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$$

is (　　).

(10%) 8.

$$A=\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

$\beta=\left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix},\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\right\}$ is an ordered basis of R^3, and $L_A: R^3\to R^3$ is the linear transformation defined by $L_A(x)=Ax$, $x\in R^3$ is a column vector of R^3, then the matrix representation $[L_A]_\beta^\beta$ of L_A is (　　).

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In each of the last two problems 9-10, not only give the answer but also provide your reasons.

(20%) 9. Find the Jordan canonical form of the matrix

$$\begin{pmatrix} 2 & -4 & 2 & 2 \\ -2 & 0 & 1 & 3 \\ -2 & -2 & 3 & 3 \\ -2 & -6 & 3 & 7 \end{pmatrix}.$$

(25%) 10. V and W are vector spaces over R (not necessarily for finite-dimensional vector spaces), and $T : V \longrightarrow W$ is a surjective linear transformation, then prove that $V \cong N(T) \oplus W$ as vector spaces, where $N(T)$ is the kernel of T.