國立清華大學 102 學年度碩士班考試入學試題

系所班組別：數學系　數學組

考試科目（代碼）：代數與線性代數 (0102)

In the problems below, Q is the field of the rational numbers, R is the field of the real numbers, and C is the field of the complex numbers.

In each of problems 1-2, just give the answer without any proof or reason.

(10%) 1. $Q(\sqrt{2},\sqrt{3})$ is the field obtained by adjoining $\sqrt{2}$ and $\sqrt{3}$ to Q, then the Galois group of this extension $Q \subseteq Q(\sqrt{2},\sqrt{3})$ is.(　　).

(10%) 2. The rank of the matrix

$$\begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$$

is (　　).

In each of problems 3-7, not only give the answer but also provide your reasons.

(16%) 3. Find the Jordan canonical form of the matrix

$$\begin{pmatrix} 2 & -4 & 2 & 2 \\ -2 & 0 & 1 & 3 \\ -2 & -2 & 3 & 3 \\ -2 & -6 & 3 & 7 \end{pmatrix}.$$

(16%) 4. V and W are vector spaces over R (not necessarily for finite-dimensional vector spaces), and $T : V \longrightarrow W$ is a surjective linear transformation, then prove that $V \cong N(T) \oplus W$ as vector spaces, where $N(T)$ is the kernel of T.

(16%) 5. Prove that an abelian group of order 30 is cyclic.

(16%) 6. Prove that a finite integral domain with unit is a field.

(16%) 7. Prove that there is no proper intermediate field between R and C, that is , prove that if E is a subfield of C and $R \subseteq E \subseteq C$, then $E = C$ or $E = R$.