類組:物理類 科目:應用數學(2001)

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※請在答案卷內作答

1. A set of three vectors in Cartesian coordinates are

$$\overrightarrow{u_1} = (1, 1, -1), \ \overrightarrow{u_2} = (-1, 1, 1), \ \overrightarrow{u_3} = (1, -1, 1).$$
 Find a set of three

vectors v_1 , v_2 and v_3 , such that their inner products

satisfying
$$\overrightarrow{u_i} \cdot \overrightarrow{v_j} = \delta_{i,j}$$
. (10%)

- 2. (a) Expand exp [tan x] up to $o(x^4)$ for |x| < 1. (5%)
 - (b) Expand $tan^{-1}x$ up to $o(x^7)$ for |x| < 1. (5%)

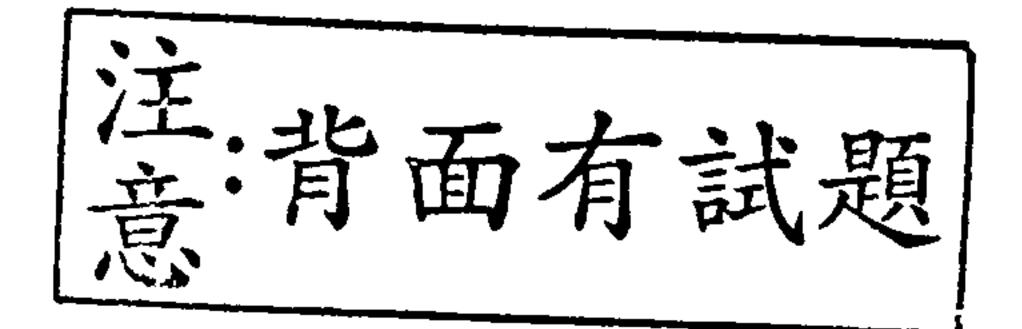
3. (a) Find
$$\frac{dI}{dx}$$
 where $I = \int_{x^2}^{\sin^{-1} x} \frac{\sin t}{t} dt$. (5%)

(b) Find the equation of tangent line to the curve

$$x^3 - 3y^3 + xy + 21 = 0$$
 at $(x, y) = (1, 1)$. (5%)

- 4. A constant density solid ellipsoid inside the surface of $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$, calculate the moment of inertia in terms of total mass for the rotation about the z-axis. (10%)
- 5. Find the solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4\sin(2x).$$
 (10%)



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6. Let us consider a functional defined by the integral,

$$I\left[x\right] = \int_{t_0}^{t_1} L\left(x\left(t\right), \dot{x}\left(t\right), t\right) dt$$

where the values t_0 , t_1 , $x\left(t_0\right)$, $x\left(t_1\right)$ are given. Here dot means the derivative with respect to t. Prove that the function $x\left(t\right)$ that minimizing/maximizing the functional $I\left[x\right]$ should satisfy

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0.$$

- (a) From the condition $\,\delta I=0$, obtain the above equation. (5%)
- (b) If the functional is defined by

$$I\left[x\right] = \int_{t_0}^{t_1} L\left(x\left(t\right), \dot{x}\left(t\right), \ddot{x}\left(t\right), \dot{x}\left(t\right), t\right) dt$$

where the values t_0 , t_1 , $x(t_0)$, $x(t_1)$, $\dot{x}(t_0)$, and $\dot{x}(t_1)$ are given. Obtain the differential equation that is satisfied by the function maximizing/minimizing the functional (10%).

7. Hermite polynomial is defined by the Rodrigues' formula,

$$H_{n}(x) = (-1)^{n} e^{x^{2}} \frac{d^{n}}{dx^{n}} e^{-x^{2}}.$$

(a) Using the Goursat's theorem,

$$\frac{d^{n}}{dx^{n}}f(x) = \frac{n!}{2\pi i} \int_{c} \frac{f(z)}{(z-x)^{n+1}} dz,$$



注:背面有試題

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obtain the generating function Jig(x,sig) defined by

$$J(x,s) = \sum_{n=0}^{\infty} \frac{s^n}{n!} H_n(x). (5\%)$$

- (b) Evaluate the integral, $\int_{-\infty}^{\infty} H_n\left(x\right) H_m\left(x\right) e^{-x^2} dx$. (10%)
- 8. Obtain the Fourier transform of the following functions: (5% each)

(a)
$$f(x) = \begin{cases} 1 & 0 \le |x| < 1 \\ 0 & 1 \le |x| \end{cases}$$
 (b) $f(x) = \begin{cases} x & 0 \le |x| < 1 \\ 0 & 1 \le |x| \end{cases}$

(b)
$$f(x) = \begin{cases} x & 0 \le |x| < 1 \\ 0 & 1 \le |x| \end{cases}$$

(c)
$$f(x) = \begin{cases} x^2 & 0 \le |x| < 1 \\ 0 & 1 \le |x| \end{cases}$$

$$(d) \ f\left(x\right) = e^{-x^2}$$